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Equity Valuation, Real Estate, and Introduction to Portfolio Management
Executive Summary (CFA I, 2003)

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Executive Summary (CFA I, 2003)

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Study Techniques for integrating required readings and abstracts.

Warm-up exercises - Just like you would not play a vigorous game of rugby without first warming up your body, you should not attempt to dive into your study session without first warming up your mind. The way to do this is to take the following steps when studying an article:

a. review the abstract
b. carefully read the introduction to the article;
c. skim the text of the article for main ideas paying attention to the figures, tables, and graphs;
d. carefully read the summary and conclusions of the article;
e. go back and skim the text again following steps a.-c. but now pay closer attention to subsections;
f. review the abstract again

Equity Valuation:

(note: this is an excellent starting point. As you proceed through the study material, relate each new topic back to the DDM. This is a strong technique because it will allow you to develop the logic and relieve you of the burden of memorizing unrelated concepts).

Dividend Discount Model

The theoretical basis of equity analysis is the Dividend Discount Model (DDM). Consequently, you should thoroughly understand how this model works, its strengths and its weaknesses. The model is:

$$P = D_1 / (k - g) \quad (1)$$

where P = intrinsic price, D_1 = dividend next period, k = required rate of return and g = growth rate of dividends.

Summary of Important Points:

1. Equation (1) represents the present value of an infinite stream of dividends growing at a constant rate (g) and discounted at the required rate of return (k). Equation (1) is a single-stage growth model because the growth rate is assumed not to change from now through eternity. That is

   $$P = D_0 (1+g) / (1 + k) + D_1 (1+g) / (1 + k)^2 + + D_\infty (1+g) / (1 + k)^\infty \quad (2)$$

   where $D_0 (1+g) = D_1$

2. With no growth in dividends and if the firm pays out all earnings as dividends, Equation (2) equals

   $$P = E_1 / k \quad (3)$$

   where E_1 is next period earnings per share

   In this case, k equals E/P or the reciprocal of the P/E ratio

3. P is not necessarily the current market price. Remember that P is your <u>estimate</u> of the intrinsic value of the stock: it is what you, as the analyst, think the stock <u>should</u> sell for on the open market. Every analyst could, and most likely will, have a unique estimate of P depending on their estimates of D_1, k and g. As we will see later, the intrinsic value equals the current market price if the market is efficient.

4. D_1 comes from projected earnings for the next period that, in turn, come from projected sales that, in turn, are generated by assets that, in turn, are financed with liabilities and owners equity. Ratio analysis helps in evaluating these interactions.

5. g is an <u>estimate</u> of sustainable growth in dividends in the projected period; it comes from the sustainable growth model: g = ROE x rate of retention. According to this model, the growth rate of dividends also equals the growth rate of sales, the growth rate of assets, the growth

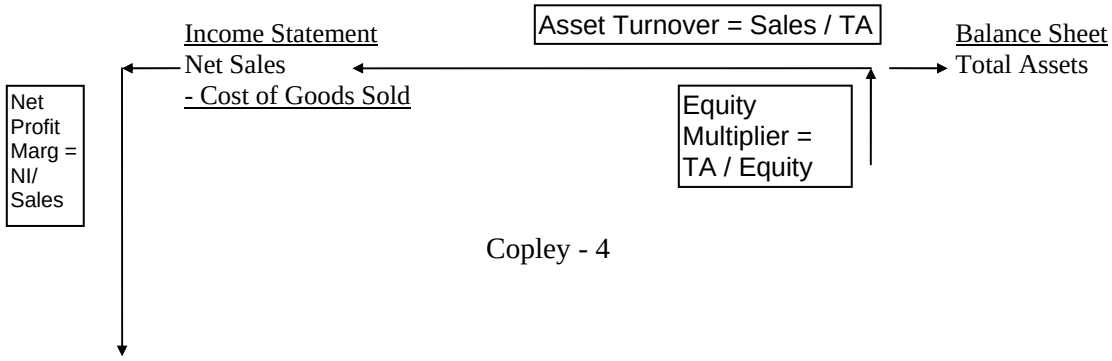
rate of retained earnings, and the growth rate of earnings if all the ratios (asset turnover, net profit margin, the rate of retention, the payout ratio, and the debt/equity ratio) remain constant in the projected period. This is why g is referred to as "sustainable." If any of these ratios change, g will likewise change. Be sure to recognize that sustainable growth is an estimate or a projection of future growth. This does not mean that sustainable growth will equal actual growth. It may or may not, again depending on whether the ratios remain constant. In other words, the estimate of sustainable growth is only as good as the assumptions that all the ratios, which show the income statement and balance sheet interrelationships, will remain constant. If you, as the analyst, believe that these interrelationships will change, you would adjust your estimate of g accordingly.

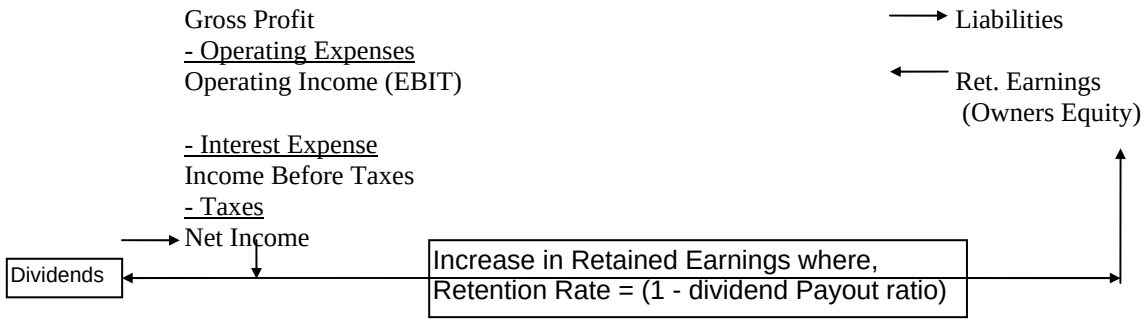
6. k usually comes from the Capital Asset Pricing Model (CAPM). Remember that this is an equity model. Thus, you must use of the required rate of return to the equity holders, not the weighted average cost of capital that reflects the required rate of return to all sources of capital including equity, debt and preferred stock. Because the CAPM is inexact due largely to using historical data to project the future, estimates of k may contain a large margin of error (standard error of the estimate in statistics). Alternatively, k could come from a sample of comparable stocks calculated as $k = D_1/P + g$ where D_1/P equals the dividend yield and g equals sustainable growth.

7. Required rate of return vs. expected rate of return: Be sure to recognize the difference between the required rate of return, which comes from some theoretical model such as the CAPM, and the expected rate of return that you project. The expected rate of return could come from a P/E multiple model (i.e., multiple projected P/E x projected earnings per share), a P/Sales multiple model, a P/Cash Flow multiple model or any other model, for that matter. The market usually attaches a high P/E multiple to a high growth stock in anticipation of the high growth, but this also implies high risk. Remember that k must exceed g for the single-stage model to make sense. For a short period of time like 3 to 5 years, g could exceed k but reason dictates that the high growth will eventually level off. Thus, some analysts use a 2-stage or even a 3-stage growth model to allow for super-normal growth.

Dividends, D_1.

Let's now take a closer look at where each of the 3 variables on the right hand side of equation (1) come from and illustrate with an example. Again, be sure to recognize the interaction between the income statement and the balance sheet. The key is retained earnings, which acts as a cumulative scoreboard for the success of the firm since day 1. To keep things as simple as possible, let's assume that all of the owners equity is accounted for by retained earnings. To see how the firm generates income to pay dividends, we need to understand the interaction of the firm's 2 financial statements: the balance sheet and the income statement. Let's begin with Total Assets on the balance sheet and trace through the process of how the firm uses assets to generate income, earnings, dividends and retained earnings.





In order to trace through the interaction of these 2 financial statements, begin with assets at time (t=0).

t = 0 (current time period)

1. Assets = $1 million
2. Net Sales = $1 million
3. Net Profit Margin (NPM) = 10%
4. Payout Ratio (PO) = 50%
5. Equity Multiplier = 2, or Debt/Equity Ratio (D/E) = 1
6. Asset Turnover (ATO) = 1

If the Equity Multiplier = 2, we know that TA = 2 and Equity = 1. Because of this, we further know that Debt (D) must equal 1. Thus, the D/E ratio must equal 1. The firm's balance sheet and income statement appear as follows:

Income Statement		Balance Sheet	
Net Sales	$1,000,000	Total Assets	$1,000,000
		Liabilities	$500,000
Net Income	$100,000	Owners Equity	
		Retain. Earnings	$500,000

At the beginning of t = 0, the firm has Total Assets of $1 million financed with $500k debt and $500k equity. At the end of the period, the firm has generated Net Sales of $1 million and Net Income of $100,000, which is then divided between dividends of $50,000 (Payout Ratio = 50%) and Retained Earnings of $50,000 (Retention Ratio = 1 - Payout Ratio). Because the firm generates $1 million sales from $1 million worth of assets, its ATO ratio is 1 (assuming ATO is calculated using ending sales and beginning assets). Note that ATO is a measure of efficiency. More sales generated from the same level of assets means greater efficiency. Another factor affecting ATO is the possibility that the firm has control over the pricing of its products. If this is the case, the firm can raise prices without increasing efficiency and, thus, raise its ATO.

Before proceeding, let's practice calculating financial ratios using two CFA I questions from the 1995 exam.

Back to our example and continuing through the loop. Retaining $50,000, money that belongs to the stockholders, allows the firm to borrow another $50,000 since we have assumed a constant D/E ratio of 1:1. Consequently, the firm now has a total of another $100,000 with which to buy more assets. At the end of period (t = 0), total liabilities and equity equals $1,100,000 and total assets equal the same $1,100,000 since the ATO equals 1. After that, the process begins again.

In review. Assets generate sales, sales generate net income, net income generates incremental retained earnings, greater retained earnings allows greater borrowing capacity (assuming a constant D/E ratio), greater debt combined with greater equity allows purchase of more assets, greater assets generate greater sales, and so forth. From this model, you can see how the firm grows--again, the key is retained earnings and the assumption is no new external equity financing.

Now, at time (t = 1), assuming constant relationships between the income statement and the balance sheet, can you calculate the growth in dividends?

t = 1 (next time period)

1. Net Sales = $1.1 million (the firm now has $1.1 million assets since we have assumed a constant ATO)
2. Net Profit Margin (NPM) = 10%, which leads to Net Income of $110,000
3. Payout Ratio (PO) = 50%, which allows payout of dividends of $55k and an increase in retained earnings of another $55k totaling $550,000
4. Equity Multiplier = 2, or Debt/Equity Ratio (D/E) = 1, which allows debt to increase to $550,000 and total asset to $1.1 million
5. Again, the Asset Turnover (ATO) = 1

Income Statement		Balance Sheet	
Net Sales	$1,100,000	Total Assets	$1,100,000
		Liabilities	$550,000
Net Income	$110,000	Owners Equity (Retain. Earnings)	$550,000

If you continue this process to t = 2, you will have net sales of $1.21 million, net income of $121k, retained earnings of $605k, debt of $605k, and total assets of $1.21 million.

Question: What is the growth rate of this firm?
Answer: 10%

Between t = 0 and t = 1, for example, sales grew from $1 million to $1.1 million (growth = 10%), net income grew from $100k to $110k (growth = 10%), total assets grew from $1 million to $1.1 million (growth = 10%), debt grew from $500k to $550k (growth = 10%), retained earnings grew from $500 to $550 (growth = 10%), and dividends grew from $50k to $55k (growth = 10%). This firm can, therefore, sustain a growth rate in dividends of 10% as long as all of the income statement and balance sheet relationships remain constant.

Sustainable Growth

	t = 0	t = 1	Growth Rate
Sales	$1 million	$1.1 million	10%
Net Income	$100k	$110k	10%
Total Assets	$1 million	$1.1 million	10%
Liabilities	$500k	$550k	10%
Owners Equity (R.E.)	$500k	$550k	10%
Dividends	$50k	$55k	10%

Now instead of going to all of this trouble in calculating g, we have a much easier method that is based on the rate of retention and the return on equity. Specifically,

g = ROE x rate of retention(4)
where ROE = net income / equity

Obviously, equation (4) is much easier to work with than tracing through all the relationships between the income statement and the balance sheet. The result, however, is the same. Remember that when working with the DDM, you are interested in projecting growth of dividends. It just so happens that growth analysis using the sustainable growth model leads to the assumption of constant growth for sales, net income, total assets, net income and equity (book value) in addition to allowing us to calculate the growth of dividends. The reason this happens is due to the assumption that the balance sheet and income statement relationships remain constant.

The question is: Is this realistic? To answer this question, let's look at projected growth rates for U.S. Healthcare, Inc. as of the beginning of 1995. Using a Value Line report (see enclosed), you can see that the growth rates in sales, dividend and earnings all differ ranging from a low for sales of 17 percent to a high for book value of 27.5 percent. The analyst apparently does not believe the relationships between the firm's balance sheet and income statement will remain constant in the future. To see this, let's go back to the above example and change one input. We will come back to the analysis of U.S. Healthcare later.

Let's assume that management changes the debt/equity ratio in (t=1) from 1:1 to 2:1--meaning the equity multiplier equals 3:1--but keep everything else the same. To get the ball rolling, let's keep the analysis for (t=0) the same also. At (t=1), after paying dividends of $50k and retaining $50k of the $100k net income earned at (t=0), equity will equal $550k, debt will equal $1,100k, and total assets will equal $1,650k. If the ATO remains constant at 1, sales at (t=1) will equal $1,650k and net income will equal $165k at a net profit margin of 10%. At a payout ratio of 50%, the firm will pay dividends of $82.5k and the process begins again for (t=2). Let's stop here and look at the growth rates as illustrated in the following table.

	t = 0	t = 1	Growth Rate
Sales	$1 million	$1.650 million	65%
Net Income	$100k	$165k	65%
Total Assets	$1 million	$1.650 million	65%
Liabilities	$500k	$1,100k	120%
Owners Equity (R.E.)	$500k	$550k	10%

Dividends	$50k	$82.5k	65%

In this example, management's decision to change the firm's capital structure (liabilities grow by 100%) leads to sales, net income, total assets and dividends all growing by 65% while equity and debt grow by 10% and 120%, respectively. Practically, the assumption of unequal growth rates is not unusual as we saw with U.S. Healthcare. Quite the contrary, it would be highly unusual for an analyst to expect equal growth rates across the board. The example shows that if any of the financial relationships change from (t=0) to (t=1), the estimate of sustainable growth in (t=0) will not materialize in (t=1). In this case, the sustainable growth rate estimate in (t=0) of 10% growth in dividends did not actually occur in (t=1). In fact, dividends grew by 65% in (t=1) due to the change in the firm's capital structure--it borrowed more money.

This bring us to another question, which is: How can the firm change growth in the future? A quick look at equation (4) shows that the firm can change g by either changing ROE or by changing the rate of retention. Careful: Changing g does not necessarily mean changing the intrinsic price of the stock, P. Why? Because the interrelationships among all 3 variables on the

Selected data from Valueline report dated January 6, 1995 by Vik Malhotra

US Healthcare, Inc (USHC)

Recent price = $40

P/E Ratio = 15.2 (Trailing 17.2, Median = 22.0)

Relative P/E = 1.11

Dividend Yld = 2.2%

Timeliness = 2

Safety = 3

Beta = 1.55

Float approximately 15% with spikes higher in times of price volatility

Institutional ownership = 118’154,000 shares

Shares outstanding = 159,920,000 (100% of capital)

Capital structure: all equity

Projected Growth rates over next 5 years:

- Revenues = 17.0%
- Cash Flows = 12.5%
- Earnings = 13.5%
- Dividends = 27.0%
- Book Value 27.5%

Projected Quarterly dividends: $.88 (1995)

Projected Earnings per share $2.85 (1995)

Projected (1997-1999) % Earned New Worth = 28%

Projected (1997-1999) % Retained to Eommon Equity = 20%

Projected (1997-1999) % All Dividends to Net Profit = 29%

Projected (1997-1999) P/E = 18

right hand side of equation (1) cause unknown effects on P. So before we look more closely at ROE and ways we can change it, let's examine these interrelationships.

Interrelationships.
If we assume no interrelationships, an increase in any one of the 3 variables on the right hand side of equation (1) would have the following impact on P:

Firm Policy	Impact on P (ceteris paribus)
Increase Dividends	Increase P
Increase growth	Increase P
Increase k*	Decrease P

* Restructuring (more debt) usually means greater risk that is reflected in a higher beta that, in turn, increases k.

Unfortunately, the assumption of no interrelationships, where each of the 3 variables acts independently of the other variables, is not realistic. For example, an increase in dividends causes a decrease in g, that leads to a decrease in k. The net effect of these interactions is often impossible to determine. This means that all you can do is make your best guess as to what impact a change in one of the 3 variables on the right hand side of the equality sign in the DDM will have on P.

The pros and cons of dividends and the effect of dividends on stock prices using the constant growth dividend discount model:

A. Arguments for greater dividends are:

1. **Clientele effect**: Presumably, investors such as retirees have some minimum income need that dividends satisfy. The higher the dividend, the more these investors are attracted to the stock. In other words, the firm's dividend policy attracts a certain type of investors. The corollary is that a cut in dividends is bad news to these investors and results in a declining stock price.
2. **Certainty of dividends versus capital gains**: This is the "bird in hand" argument that says cash dividends are more definite than capital gains and, thus, less risky. Therefore, the higher the dividends, the lower the discount rate, k.
3. **Tax effects**: Tax exempt investors such as pension funds, endowments and foundations pay no income tax on dividends. Thus, they prefer higher dividends opposed to lower dividends.

Arguments against higher dividends are:

1. **No effect on value**: The firm's investment in assets (plant and equipment--left hand side of balance sheet) determines its profitability, not the method of financing (debt and equity--right hand side of balance sheet). Actually, this is the Modigilani and Miller (M&M) argument that the firm's capital structure is irrelevant. In other words, the firm's decision to invest in assets is independent of the manner in which it finances those assets. Thus, the firm's dividend

policy is independent of the firm's value. Another way of looking at the M&M argument is that the firm's weighted average cost of capital (WACC) is constant. More on this point below.
2. **Less cash available for investments**: Dividends reduce the amount of cash the firm has to finance profitable investments. This argument is the "pecking order" argument that says the firm wants to use internally generated funds (retained earnings) before it uses externally generated funds (new stock or bond issues).
3. **Taxes**: Investors must pay taxes on dividends, but not on appreciation--at least not until the time of sale. Capital appreciation represents a deferral of taxes.
4. **Violation of an indenture**: Many bond indentures limit the amount of dividends a firm may pay in an effort to afford the bond holders greater protection.

B. Holding all other factors constant, an increase in dividend payout would

(i) decrease the firm's growth rate. You can see this from equation (4). A greater dividend payout ratio means a lower retention rate. Lower growth could lead to a lower stock price to the extent that lower growth leads investors to attach a lower P/E multiple to the stock.

(ii) decrease growth in the firm's book value because the firm would have less internally generated funds (retained earnings) with which to finance the acquisition of new assets it needs for growth.

Above, I mention that the firms would not want to pay out higher dividends because, according to M&M, the firm's weighted average cost of capital is constant. This theory says that if the firm wants to invest in an asset, it makes no difference to the value of the firm's stock if the firm finances this investment with all debt, all equity, or some combination of the two.

Assume, for example, that the firm has $1 million of net income that it could either pay out as dividends or retain. Simultaneously, the firm wants to invest $1 million in a new asset. The question is: How should the firm finance the purchase of the asset in order to maximize the value of the firm to the stockholders?

If the firm pays out the $1 million as dividends, then it must borrow $1 million in order to finance the purchase the asset. If, on the other hand, the firm retains the entire $1 million then it would be able to purchase the assets with equity since retained earnings belong to the stockholders. A third alternative is for the firm to use some combination of equity (retaining earnings) and borrowing. In any case, according to M&M's theory (**no taxes**), the value of the firm is unaffected. If the firm uses debt, which is cheaper than equity due to less risk (priority of claims on assets of firm) and the tax deductibility of interest, risk to the stockholders increases causing the cost of equity to rise such that offset the benefits of the cheaper debt. Thus, the weighted average cost of capital stays constant and the value of the firm is unchanged. If the firm uses equity (retained earning), the stockholders incur an opportunity cost of not receiving dividends, the weighted average cost of capital is again unchanged, and the value of the firm is unchanged. In either case, the firm's weighted average cost of capital and the firm's value remain constant.

Now we all know that the assumption of no taxes is unrealistic. M&M further developed their theory of capital structure (**with taxes**), and conclude that the capital structure is important due

to the tax deductibility of interest. In fact, M&M go the other direction from proposition I and say that the firm should use all debt, or as much debt as possible. Because we can easily see that this conclusion is also unrealistic, why is the M&M theory so important?

The reason is that it allows us to focus on what the theory ignores--that of bankruptcy costs. As the firm uses more and more debt, it increases the chances of bankruptcy. At some point, the chance of bankruptcy and the associated costs (legal and administrative costs, impaired ability to conduct business, and agency costs) offset the benefits of the cheaper debt. When the costs just equal the benefits, the firm is at its optimal capital structure.

Summary of theories of capital structure:

1. M&M **(no taxes)** --no optimal capital structure
   Proposition I--value of firm is independent of capital structure, and the firm's weighted average cost of capital is constant.
   Proposition II--the cost of equity rises as the firm increases its use of debt
2. M&M (**with taxes)**--the firm's optimal capital structure is 100% debt
   Proposition I--debt financing is highly advantageous due to the tax shield of interest, and the firm's weighted average cost of capital consistently decreases as the firm relies more heavily on debt
   Proposition II--the cost of equity as the firm relies more heavily on debt (same as Proposition II without taxes).

3.Bankruptcy Theory--the firm's optimal capital structure is between 0% debt and 100% debt. The optimal capital structure is where the cost of bankruptcy, which increases with the greater use of debt, equals the benefits of using lower cost debt (think of this as the economic concept of marginal cost equaling marginal revenue).

Here is a use of the WACC in valuation.

Two Equivalent Methods of Valuing a Firm's Equity

Using the discounted cash flow model (Value = Cash Flow / Discount rate - Growth), you can determine a firm's equity value in either of two ways:

1. direct method: calculate the present value of cash flows accruing to the stockholders.
2. indirect method: calculate the present value of cash flows accruing to both stockholders and creditors, then subtract the value of debt.

Once you find the equity value of the firm, divide this value by the number of shares outstanding to determine the value per share.

The main thing to remember is that whichever method you use, be sure your cash flow calculation matches your discount rate. In other words, if you calculate cash flows excluding debt (assuming the firm is unlevered), use the required rate of return to the stockholders, Ke; if you calculate cash flows including debt (assuming the firm is levered), use the weighted average cost of capital, WACC. As we will see below, both methods provide the same answer--I promise!

Method	**Discount Rate**
cash flows to stockholders	required rate of return to stockholders, Ke
cash flows to creditors and stockholders	weighted average cost of capital, WACC

Definitions:(Be careful with these definitions because other authors may use slightly different definitions)
Free cash flow (FCF) is the cash flow available for distribution after the firm invests in new plant and equipment.

Free cash flow available to stockholders = net income + depreciation - investment

Free cash flow available to both stockholders and creditors = operating cash flow - taxes + depreciation - investment

where

Operating cash flow = Earnings before interest and taxes (EBIT)+ Depreciation (D)
= Revenue (R) - Variable Costs (VC)- Fixed Cash Costs (FCC)

thus

Free cash flow available to stockholders and creditors = EBIT (1 - t) + D - Investment

Note that when growth equals zero, depreciation equals investment and that:

Free cash flow available to stockholders = net income

Free cash flow available to both stockholders and creditors = EBIT (1-t)

Be careful with taxes because the two methods calculate taxes differently. The direct method deducts interest for tax purposes whereas the indirect method does not. Remember that the indirect method values the firm's equity by discounting cash flows accruing to both stockholders and creditors before subtracting the value of debt.

Example

Calculate the value per share of the firm's stock given the following projections for next year (all projections in 000s):

Revenue (R)	$1300
less: Variable costs (VC)	-600
less: Fixed cash costs (FCC)	-0
less: Noncash charges (Depreciation, D)	-200
Earnings before interest and taxes (EBIT)	500
less: Interest (I = 10%)	-50
Earnings before taxes (EBT)	450
less: Taxes (t = 50%)	-225
Net income (NI)	$225

Assume:

Debt = $500,000

Cost of debt before taxes = Kd = 10%

Cost of debt after taxes = Kb = 10% (1 - .5)
Ke = 30%
Debt / Total Asset ratio = 40%
Equity / Total Asset ratio = 60%
Shares outstanding = 100,000
Growth (g) = 0

The no-growth assumption means that the firm pays out all earnings as dividends, and that depreciation equals investment. Although not essential, this assumption simplifies the analysis. Given the above assumptions, the WACC equals:

WACC = Kb (Debt / Total Assets) + Ke (Equity / Total Assets)
= .10 (1 - .5) (.6) + .30 (.4)
= 20%

Direct method of calculating value of equity (E):
Equity = ***Free cash flow available to stockholders /*** Ke
=$225,000 / .3
= $750,000 (or $7.50 per share)

Indirect method of calculating Equity:
Equity = Total value of firm - debt
where:
Total value of firm = ***Free cash flow available to both stockholders and creditors*** / WACC
= (500,000) (1 - .5) / .2
= 250,000 / .2
= $1,250,000 (according to this method, taxes = $250,000)

thus

Equity = Total value of firm - debt
= $1,250,000 - $500,000
= $750,000 (or $7.50 per share)

As you can see, both methods give the same answer, just like I promised.

Retention Rate impact on g.

Because of the interrelationships among the 3 variables (D_1, k, and g) impacting P, an increase in the retention rate will lead to higher growth, as you can see in equation (4), but we do not know for sure how this change will impact P. The unknown is how stockholders will view the change. If they believe that the firm will generate a risk-adjusted rate of return greater than the return they (the stockholders) could achieve for themselves, the price of the stock should increase. This is why k is referred to as an opportunity cost--it represents the stockholders' lost opportunity to invest the money management retains in another investment of equal risk. On the other hand, if the stockholders believe that they could generate a risk-adjusted rate of return greater than what management could do, the price of the stock should decline. Complicating the issue of how the price of a stock will react to a given change in D_1, k, and g is the movement of

the overall market, which is a very powerful force. Down markets can bring down a good stock and a good market can bring up a bad stock. The saying is "A rising tide lifts all ships."

As you can see, how P reacts to a change in the retention rate or, for that matter, a change in k or a change in D_1 is impossible to know before hand. Instead of trying to sort out these relationships, I suggest that you use common sense. For example, if a firm is growing quickly in a growing industry (Stage I growth), common sense suggests that a high retention rate is probably wise (i.e., a software company like Microsoft). On the other hand, if a firm is growing slowly in a mature or declining industry (Stage III growth), a low retention rate is most likely wise (i.e., an electric utility company like Duke Power). The life-cycle model of growth shown below more clearly shows the interaction between a firm's retention rate and its dividend policy. You can also see from this model how the firm's leverage, net profit margin and the level of competition in the industry change over time.

Life Cycle Model (please note that some writers prefer 4 stages instead of 3, although the basic ideas are the same)

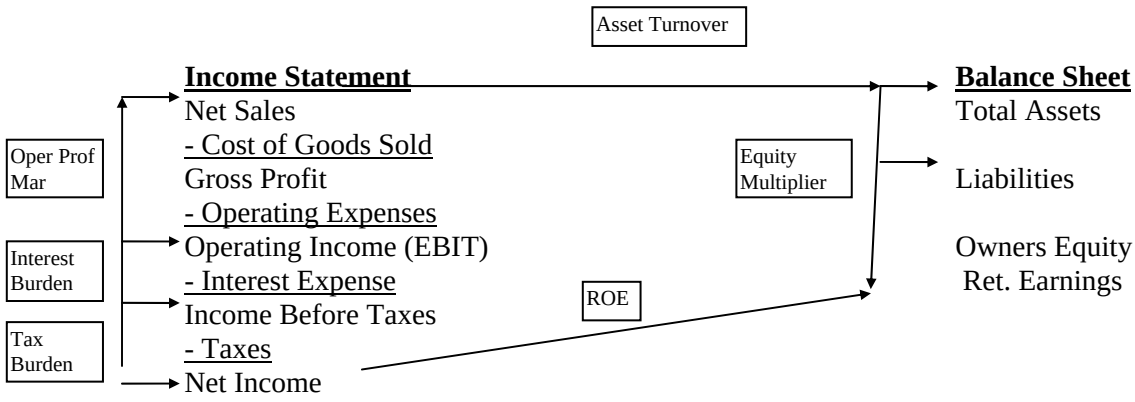
Firm Financial Policies	**Stage I: Rapid Expansion**	**Stage II: Mature Growth**	**Stage III: Stabilization or Decline**
Dividends	Low	Moderate	High
Retention Rate	High	Moderate	Low
Leverage	High	Moderate	Low
Net Profit Margin	High	Moderate	Low
Competition	Low	Moderate	High

Components of ROE.

Now, let's look more closely at ROE and its components to see how we can impact growth. By definition,

ROE = net income / equity

Alternatively, you can decompose ROE into several components--usually either 3 or 5 components--that, when multiplying all the components together, the produce equals ROE. An easy way to think about this decomposition model (without relying on memory) is to visualize the income statement and balance sheet side-by-side. Begin with net income and work your way up the income statement to net sales, leap over to total assets on the balance sheet, and finally bounce down to equity as follows:



This layout allows you to see the components more clearly.

ROE = (Net Income / Income Before Taxes) x (Income Before Taxes / Operating Income) x (Operating Income / Net Sales) x (Net Sales / Total Assets) x (Total Assets / Equity)

where:

Net Income / Income Before Taxes = Tax Burden
Income Before Taxes / Operating Income = Interest Burden
Operating Income / Net Sales = Operating Profit Margin (OPM)
Net Sales / Total Assets = Asset Turnover (ATO)
Total Assets / Equity = Equity Multiplier

Tax Burden. You will note that as taxes increase, the Tax burden becomes smaller (the ratio Net Income / Income Before Taxes approaches zero); and that as taxes decrease, the Tax Burden become larger (the ratio Net Income / Income Before Taxes approaches one). This is somewhat confusing, I realize, but it is just a result of terminology.

The same thing happens with the Interest burden. That is, when interest increases, the Interest Burden becomes smaller (the ratio Income Before Taxes / Operating Income approaches zero); and when interest decreases, the Interest Burden becomes larger (the ratio Net Income / Income Before Taxes approaches one). In other words, a Tax Burden ratio of 1 would indicate that the firm is paying no taxes. Question: Could the Tax Burden ratio be greater than 1? Yes, it could if the firm has tax losses from a prior year that result in a tax refund. It could also be negative if the firm has negative net income and positive Income Before Taxes.

Interest Burden. This model also allows you to see the impact on the income statement from increasing debt in the capital structure of the firm (right hand side of the balance sheet). You may think that increasing debt would lead to a greater interest expense that, in turn, would decrease the Interest Burden (the ratio approaches zero as the interest expense increases). This may or may not happen because interest rates in the economy may be decreasing at the same time the firm is increasing debt. The combination of more debt and lower interest rates could lead to a decline in the Interest expense on the income statement that, in turn, increases the Interest Burden (the ratio approaches 1). My point is that the interest rate the firm pays determines the interest expense on the income statement, and that this rate can be volatile due to

market forces. Thus, you cannot say with certainty that increasing debt leads to an increase in the interest expense unless you know more about the direction of interest rates in the overall economy.

Moreover, when a firm increases debt (leverage), the interest rate it must pay depends on the maturity of the debt security it uses. For example, if the firm employs a large amount of short-term debt, whose rates are very volatile, the interest burden could decrease significantly as the firm rolls over maturing debt into new, higher interest rate debt. Alternatively, if the firms employ a large amount of long-term debt, whose rates are not very volatile, the interest burden may not change much at all as the firm increases leverage. An economist would say that the yield curve is sticky on the long end, meaning that short-term rates are more volatile than long-term rates. Remember the matching principle: finance short-term assets with short-term debt securities, and long-term assets with long-term debt securities. By not following the matching principle (i.e., financing long-term assets with short-term debt), the firm exposes itself to the risk that short-term interest rates may increase just as the firm rolls over the short-term debt. This is reinvestment risk. My point is that you cannot determine the impact of increasing debt (balance sheet item) on the firm's interest expense (income statement item) unless you know more about the maturity structure of the debt instruments as well as the direction of interest rates in the economy.

Operating Profit Margin. At first blush, you would think that an increasing OPM is the result of cost containment, which is an example of greater efficiency. This may be true, but it does not tell the whole story. An increasing OPM could come from the firm's ability to raise the price of its product quicker than costs are going up. Competitive forces within the industry, in turn, impact this ability. If, for example, the firm is in the early stages of growth (Stage I), the firm may be relatively free in setting prices due to the lack of competition. In the mature stage of growth (Stage III), competition has increased meaning that the firm is now facing thin profit margins and is not free in setting prices. My point here is that an increasing OPM may come from either greater efficiency generated from cost containment, or it could come from the firm being a leader in the industry with the ability to set prices, at least in the short run. Over the longer term as the industry matures, that pricing ability may slip as the industry becomes more competitive.

Equity Multiplier. In addition to the interaction between leverage (balance sheet item) and interest expense (income statement item) discussed above, an important factor affecting ROE is the impact of the Equity Multiplier. The key issue is whether the firm can generate a return on the borrowed money greater than the interest rate it must pay. A common sense approach to this concept leads to the conclusion that the firm must earn a before-tax return on the assets it purchases with the borrowed money greater than the after-tax rate it pays for the money. Using more specific finance terms, the After-tax Return on Assets, ROA_{at}, must exceed the interest rate the firm pays its creditors after adjusting for taxes. The equation is:

$$ROA_{at} > i\,(1 - t)$$

where: ROA_{at} = Operating Income (1 - tax rate) / Total Assets

i = the interest rate on borrowed money

t = the firm's tax rate

Example. It is easier to see this relationship if you think of a new firm (t = 0). Assume a firm purchases $100 worth of assets by borrowing $50 (debt) and having the stockholders invest $50 (equity). If the firm generates Operating Income of $10 and is in the 40% tax bracket, it has after-tax operating income of $6 on a $100 investment, for an ROA_{at} of 6%. In this case, the interest rate after taxes is also 6% [10% x (1 - .4)] with the income statement and balance sheet appearing as follows:

Example A. With leverage where $ROA_{at} = i (1 - t)$:

Income Statement		**Balance Sheet**	
Operating Income	$10	Total Assets	$100
- Interest ($50 x 10%)	-$5		
Income Before Taxes	$5	Liabilities	$50
- Taxes (@ 40%)	-$2		
Net Income	$3	Owners Equity	$50

After-tax interest rate = 10% (1 - .4) = 6%
and,

ROA_{at} = $10 (1 - .4) / $100
= 6%

Since ROAat equals the after-tax interest rate, the use of leverage does not impact ROE one way or the other. We can see this by observing ROE in Example A is

ROE = $3 / $50
= 6%

If we had not used any leverage, the income and balance sheet would appear as follows:

Example B. Without leverage where $ROA_{at} = i (1 - t)$:

Income Statement		**Balance Sheet**	
Operating Income	$10	Total Assets	$100
- Interest ($50 x 10%)	-$0		
Income Before Taxes	$10	Liabilities	$0
- Taxes (@ 40%)	-$4		
Net Income	$6	Owners Equity	$100

ROE = $6 / $100
= 6%

which is the same as with leverage. My point is that unless ROAat exceeds the after-tax cost of debt, the use of leverage will not increase ROE and, thus, will not increase growth.

One final example: let's see what happens if ROA_{at} does exceed the after-tax interest rate as follows:

Example C. With leverage $ROA_{at} > i (1 - t)$:

Income Statement		**Balance Sheet**	
Operating Income	$15	Total Assets	$100
- Interest ($50 x 10%)	-$5		
Income Before Taxes	$10	Liabilities	$50
- Taxes (@ 40%)	-$4		
Net Income	$6	Owners Equity	$50

After-tax interest rate = 10% (1 - .4) = 6%
and,

ROA_{at} = $15 (1 - .4) / $100
= 9%

Now,

ROE = $6 / $50
= 12%

Our conclusion is that the use of leverage increases ROE that, in turn, increases g (assuming a constant retention rate) as long as the after-tax return on assets exceeds the after-tax interest rate. If , ROA_{at} is less than the after-tax interest rate, ROE would decline and this would negatively impact growth.

Back to the decomposition of ROE: The benefit of decomposing ROE is that it allows you to analyze the impact each component has on ROE and, thus, on growth. To illustrate, the Merck question below shows that ROE increased from 20.7% in 1985 to 42.8% in 1990. The question is: What caused this increase? By decomposing ROE into its 5 components, the question clearly shows that greater use of leverage (reflected in the higher equity multiplier) and increased efficiency (reflected in a higher ATO) were the major reasons. Without decomposing ROE, you would not be able to detect these influences.

The Required Rate of Return, k.

Thus far, we have discussed how the firm generates dividends, and the meaning of sustainable growth. The last variable on the right hand side of the equality sign that determines the intrinsic value of a stock, P, in the DDM is k. Let's now see where k comes from. The required rate of return to the stockholders, k, comes from the Capital Asset Pricing Model (CAPM), which several theoreticians developed as an extension of the Fisher equation. Professor Fisher developed this model in 1930 and it has laid the foundation for much of Modern Portfolio Theory (MPT). The Fisher states that the required rate of return, k, equals a risk-free rate plus a risk premium as follows:

k = risk-free rate + risk premium

Risk-free Rate.: Many people use the 90-day Treasury-bill (a 90-day zero) rate as the risk free rate. This is acceptable only if the investor's time horizon is 90 days. If it is not, then the maturity of the zero should coincide with the investor's time horizon. Thus, if your time horizon is long-term--like 20 years--your choice of a risk-free security should be a 20-year zero-coupon Treasury bond. If, instead, you chose a 90-day T-bill, you would continually have to roll over the T-bill every 90 days at unknown rates. As a consequence, you would not have a risk-free rate due to reinvestment risk.

The risk-free rate actually has two components: the real rate of return and an inflation premium. Be careful here because many people get this very confused. You must remember that we are dealing with expectations, not observations. That is, the real rate is not observable, nor is inflationary expectations. Thus, we must estimate both.

As an observation (ex post), the real rate equals the risk-free rate minus the rate of inflation. As an expectation (ex ante), the real rate depends on growth of the labor force, growth in the number of hours worked per week, and the growth in labor productivity. If, for example, 100 workers make 100 widgets in year 1, and the same 100 workers make 103 widgets in year 2, the real rate of interest is 3 percent--assuming the same number of hours worked. An economist would say that the 3 percent is a reward for postponing consumption.

Remember that the required rate of return is forward-looking (ex ante), not backward-looking (ex post). Thus, you cannot use the historical real rate as an expected real rate unless you make the heroic assumption that the future will look exactly like the past. The problem with this assumption is that it does not allow for increases in productivity that result from technological advances, which tend to occur rather slowly. In the U.S., some economists estimate the real rate of return to be around 2 - 2.5 percent. To this real rate, you then must add a premium for expected inflation, which is also unobservable. As you can see, the risk-free rate in the CAPM causes a problem. Still, as mentioned above, many people use the T-bill rate for convenience.

Risk Premium.: Now, let's discuss the risk premium, which led to development of the CAPM. To do this, we will begin with development of the Markowitz efficient frontier (a curve) and proceed to an intuitive expansion of that model to the Sharpe efficient frontier (a straight line). The main conclusion of the Sharpe model is that all investors will invest in the market portfolio, M. Using this conclusion, we will intuitively show that beta measures the risk of a security relative to the risk of M. When we multiply this relative risk (beta is a measure of micro risk), by the excess of the expected return of M over Rf (a measure of macro risk premium) we get a measure of the total risk premium in the Fisher equation. I believe this is plenty, for now.

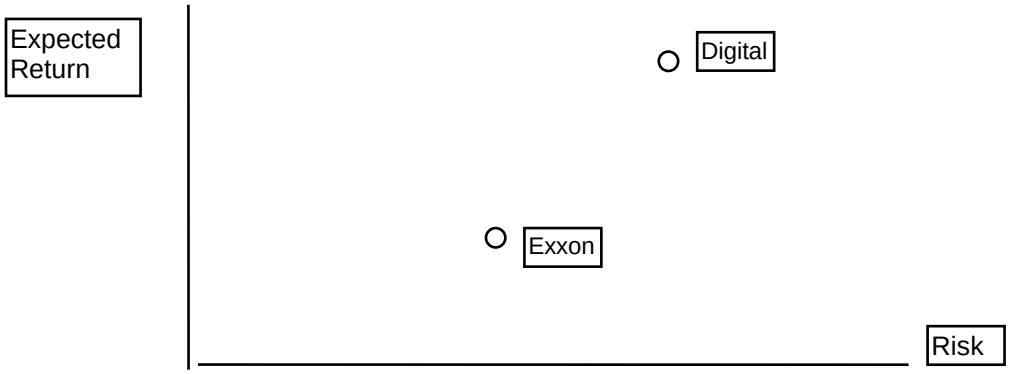
Markowitz Model.

To begin the development of the Markowitz efficient frontier, let's assume that we have only 2 securities with expected return and risk (standard deviation = SD) as follows (see Investments, Bodie, Kane and Marcus, Chapter 7, 1989 for more detail; you can also find similar treatment with different numbers in Chapter 7 of the 1993 edition of Investments):

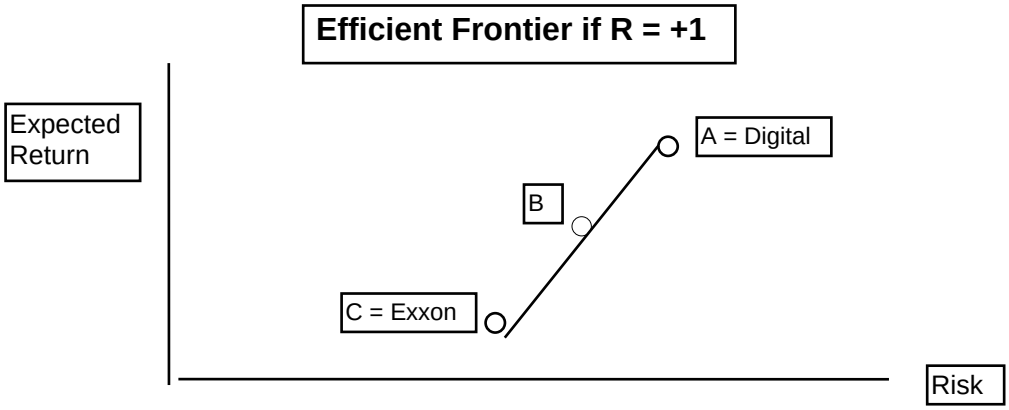
	Digital	Exxon

Expected Return, E(r)	.20	.15
Standard Deviation, SD	.45	.32

The graph below shows a plot of these data.



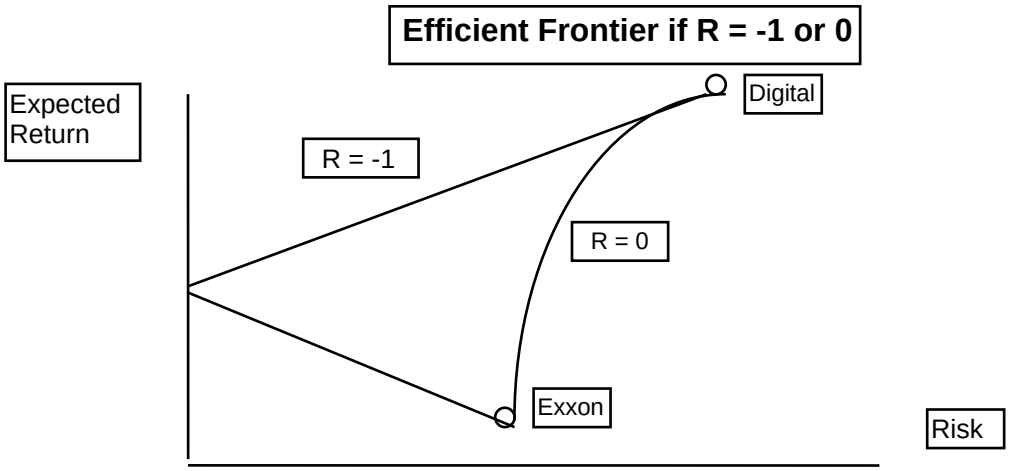
The question is: What does the efficient frontier look like for these 2 securities? Is it a straight line connecting the 2 points, or is it a curve? The answer depends on the correlation between Exxon and Digital. If the correlation coefficient, R, is +1, then the efficient frontier is a straight line as shown below



The table below shows portfolio expected returns for 3 different allocations of your money. (When we expand our thinking to include other asset classes such as bonds, internationals and real estate, we will use this same approach in developing the Markowitz efficient frontier.)

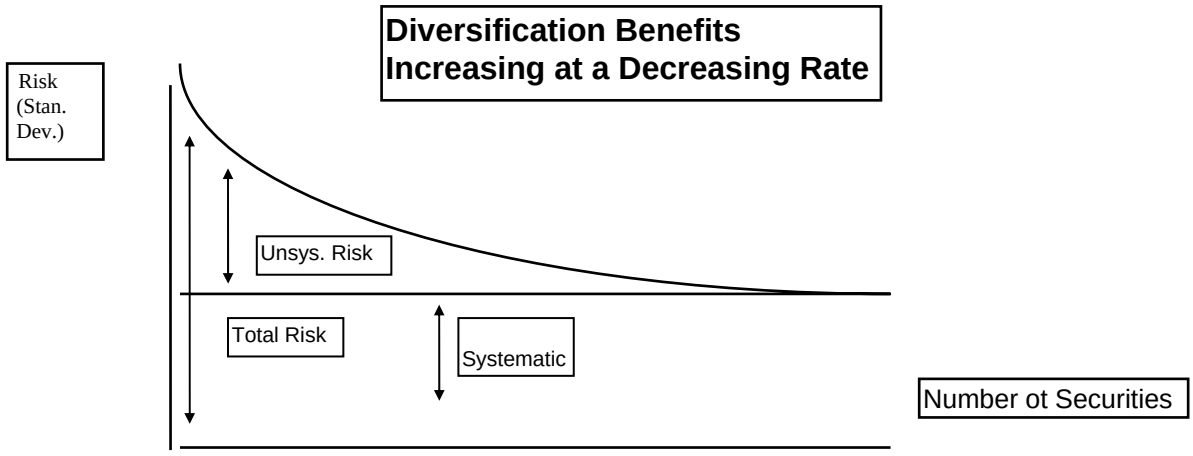
	Allocation (Digital / Exxon)	E(Rp)	SDp
Point A	100% / 0%	.20	.45
Point B	50% / 50%	.175	.385
Point C	0% / 100%	.15	.32

But what if R does not equal +1, perhaps something less than +1? What will the efficient frontier look like then? Any combination of 2 securities whose R is less than +1 generates diversification benefits, which is the key concept of Modern Portfolio Theory. If, for example, R = 0, the efficient frontier bows out and to the left as shown in the following graph:



You can see from this graph that the efficient frontier becomes more efficient as R approaches its lower limit, which is -1 (theoretically possible, but not very practical). Most people would say that an average R value (the average of all possible pairwise combinations of all securities in the world) of around .3 is a reasonable guess.

The next question is: What happens to the efficient frontier as we add more securities to the portfolio? The graph below shows that total risk of the portfolio, as measured by the standard deviation, becomes smaller as you continue to add more securities but that it will not go below systematic risk. Moreover, note that total risk decreases at a decreasing rate. This means that diversification benefits are greater when adding the 10th security than when adding the 110th security. Even stated another way, the efficient frontier continues to bow up and to the left as you add more an more securities to the portfolio, but the increments at which it moves become smaller and smaller with the addition of each new security. This is a very important point so be sure you understand it. Because perfect diversification is possible only in theory, we need a workable model and the concept of diversification benefits increasing at a decreasing rate provides such a model. The practicality of the model is that you need only around 15 to 20 securities to achieve good diversification.



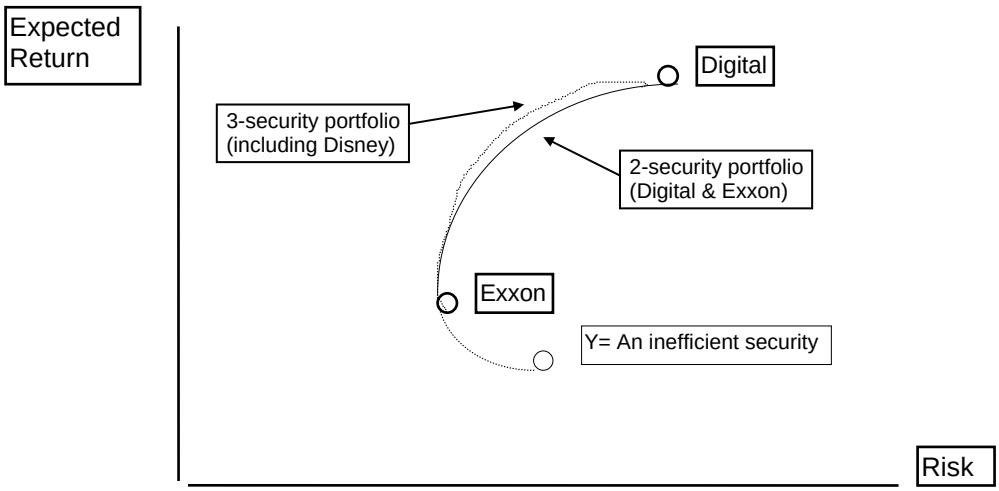
The above graph says that increasing the number of securities in the portfolio decreases the total risk at a decreasing rate. This is an important concept and you will see it again in the efficient frontier. That is, as you add more and more securities to the portfolio, the efficient frontier bows out and to the left at a decreasing rate. Stated differently, securities 4 and 5 added to the portfolio reduce total risk more than securities 100 and 101.

Let's now return to our 2-security portfolio (Digital and Exxon) to see how to construct the final efficient frontier that includes all securities in the market. With only 2 securities and a correlation coefficient, R, of approximately .3, we saw that the efficient frontier is a curve. Let's add a third security (perhaps Disney) for diversification purposes. This is a little tricky, so be careful. When adding a third security to an existing portfolio of 2 securities, you are interested in the R between the third security and the combination of the first 2 securities, which now acts as one security since we have already combined them into a portfolio. In other words, the third security generates an R that measures the correlation between Disney and the 2-security portfolio of Digital and Exxon. Understand?

Note: If you were evaluating the entire universe of stocks to find 2 stocks in which to invest, you would, theoretically, check all possible R's for all possible 2-security combinations. Having done

this, you would select the 2 with the lowest R (In the above example, I just arbitrarily said these 2 were Digital and Exxon).

Having constructed a 2-security portfolio, you would then check all possible R's between this 2-security portfolio and a third security in order to select the next security that gives you the most diversification benefits--the security that makes the efficient frontier bow out and to the left as far as possible. The 3-security efficient frontier of Digital, Exxon and Disney appears as follows:



Having constructed the 3-security efficient frontier, you would follow the same procedure in selecting a fourth security, a fifth security, and so forth. As you add more and more securities to the portfolio, each additional security provides incrementally less diversification benefits. In other words, diversification benefits increase at a decreasing rate.

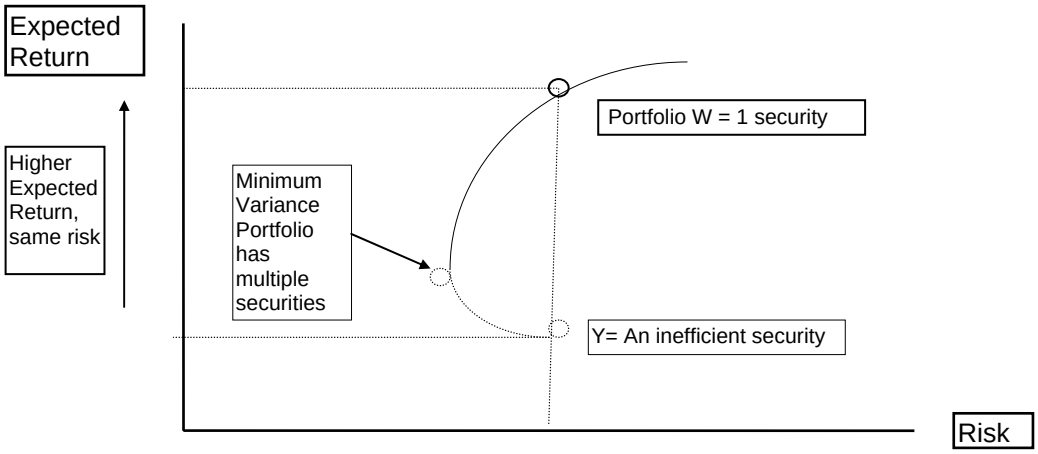
Let's go back to the 3-security efficient frontier for just a minute. In the above graph, the upper end point represents the highest expected return and highest risk security of the three securities you are analyzing. The lower end point represents the lowest expected return and lowest risk security. But what if you have an inefficient security like Y (perhaps a gold mining stock) in the above graph? Would you include Y in the construction of the efficient frontier as a 4th security even though it has higher risk and lower expected return than Exxon. That is, Y is inefficient. The answer is a qualified yes: depending on the correlation between Y and the existing 3-security portfolio of Digital, Exxon and Disney.

Let's assume that Y has low correlation with the existing 3-security portfolio and, as a consequence, including it in the construction of the efficient frontier as a fourth security provides positive diversification benefits. As you can see from the graph, Y makes the frontier bend backward. Therefore, we need to know when to invest in Y and when not to invest in Y. But first, we need a definition.

Minimum Variance Portfolio.
After including any inefficient securities like Y in the construction of the efficient frontier, the point where the frontier begins bending backward is called the minimum variance portfolio

(MVP). For practical purposes, we can call the MVP the lower end point of the efficient frontier. Unlike the upper end point that contains only one security, however, the lower end point (MVP) contains more than one security. In our example, it includes portions of Digital, Exxon, Disney and Y. We now know when to invest in a security like Y: you would do so in order to generate positive diversification benefits for the portfolio as a whole. Investing in the minimum variance portfolio, which contains Y, is rational for the investor who wants to minimize his or her risk but achieve an expected return greater than the risk-free rate. Careful: remember that due to the positive diversification benefits of investing in Y, all portfolios to the right of the minimum variance portfolio also contain portions of Y except the upper end point.

Now that we know when to invest in an inefficient security like Y, what about the question of when not to in an inefficient security? As we just saw, you want to invest in inefficient securities for diversification purposes. This is a portfolio perspective. From an individual security perspective, however, you would not want to invest in inefficient securities Y. You can see this in the graph below that shows you can achieve a higher expected return with the same level of risk by investing in portfolio W opposed to security Y.



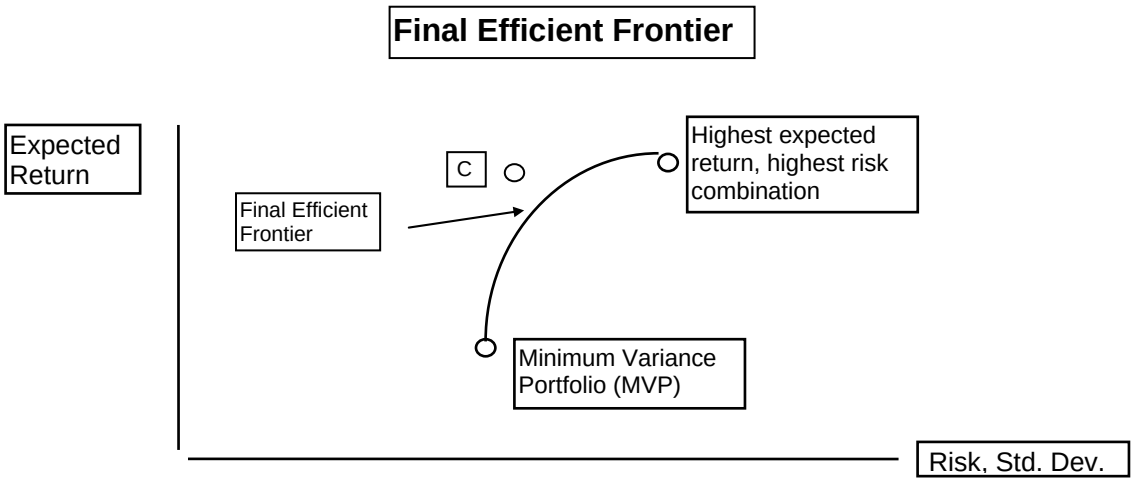
Until now, we have been only dealing with stocks. Later, we will expand the concept of the efficient frontier that includes all other asset classes such as bonds, real estate, and internationals, but not yet. Can you anticipate the consequences of a multiple asset-class portfolio where each asset class is less than perfectly correlated with the other asset classes? Construction of the efficient frontier for such a portfolio follows the same process we just reviewed, which depends on the correlations between and among asset classes. The shape of a multiple-asset-class efficient frontier shows that the upper end point represents the asset class with the highest expected return and the and lowest expected risk; the lower end point (MVP) may contain more than one asset class if inefficient asset classes provide positive diversification benefits. A final comment: just like we said for individual securities, the multiple-asset-class efficient frontier bows out and to the left at smaller and smaller increments as you sequentially add more and more asset classes. Now, let's get back to stocks.

Question: What is happening to the efficient portfolio as you methodically include more and more stocks in the portfolio?

Answer: It continually becomes more efficient.

Question: After repeating the process of including, theoretically, thousands or even hundreds of thousands of stocks, what is the shape of the final efficient frontier?

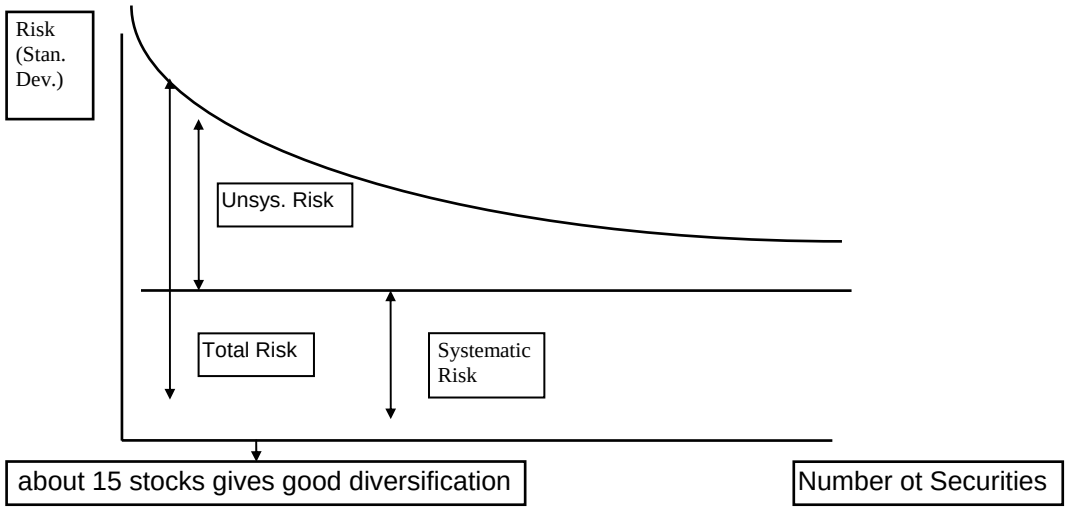
Answer: It bows up and to the left as far as the market permits. In other words, the final efficient frontier includes all stocks in the world, meaning that there are no stocks such as stock C that plots further up and to the left of the frontier (see the graph below). Stock C requires a special comment because it can, if you are not careful, cause a serious misunderstanding. Most texts state that an assumption necessary for construction of the efficient frontier is homogeneous expectations. In other words, all investors see the same big scoreboard in the sky at the same time. This assumption, then, allows you to construct only one efficient frontier like the one I drew below. Thus, stock C does not exist since it is further to the left of market expectations. If, on the other hand, all investors do not have homogeneous expectations, then each investor could construct a different efficient frontier based on his or her unique outlook for the market. In this case, a stock like C could exist. Finally, if you construct the efficient frontier using historical ex post data, then C could not exist. More on the ex post frontier below.



Question: Where does the greatest increase in efficiency occur, from adding the 3rd security to the portfolio, or adding the 100th security?

Answer: The greatest efficiency occurs from adding the 3rd security because efficiency increases at a decreasing rate. Recall that the construction process began with selecting the first 2 securities that were the least correlated of all securities in the market. We added a third security that, in turn, was the least correlated security of all other securities with the existing 2-security portfolio, and so forth. As we continued with this process, the last security added to the portfolio was the one that generated the least amount of diversification benefits. Otherwise, it would not have been the last security added to the portfolio. In other words, diversification benefits

increase at a decreasing rate as shown in the following graph. Be sure to recognize that the only portfolio that is perfectly diversified is the market portfolio, which lies to the far right of the graph (not shown).



Increasing efficiency at a declining rate is why the curve in the above graph declines at a decreasing rate, and why a good rule of thumb is that a well diversified portfolio contains approximately 15 securities even though a perfectly diversified portfolio contains <u>all</u> securities in the market.

Total Risk = Systematic Risk + Unsystematic Risk

In this equation, unsystematic risk is micro: you <u>can</u> eliminate it through diversification. Systematic risk is macro: you <u>cannot</u> eliminate no matter how many securities you include in your portfolio. Thus, perfect diversification means that you would have to invest in every possible security in the market, which is the market portfolio, M.

It is important that you see the relationship among 3 different models: (1) the way the efficient frontier bows up and to the left as you add more and more securities to the portfolio--like we did in building the final efficient frontier, (2) the way the total risk declines at a decreasing rate as you add more and more securities to the portfolio--like we did in the last graph, and (3) the mathematical equation for total risk (standard deviation of the portfolio). Carefully study the following equation and make sure you can relate it back to the final efficient frontier and the total risk graph.

$$\sigma(\text{portfolio}) = \left[\sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w_i \sigma_i \sigma_i \rho_{ij}\right]^{1/2}$$

2-Asset case: $\sigma_p = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_1 w_2 \sigma_1 \sigma_2 \rho_{12} + w_2 w_1 \sigma_2 \sigma_1 \rho_{21}$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2\, w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

3-Asset case: $\sigma_p = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma^2_3 + w_1 w_2 \sigma_1 \sigma_2 \rho_{12} + w_1 w_3 \sigma_1 \sigma_3 \rho_{13} + w_2 w_3 \sigma_2 \sigma_3 \rho_{23} + w_2 w_1 \sigma_2 \sigma_1 \rho_{21} + w_3 w_1 \sigma_3 \sigma_1 \rho_{31} + w_3 w_2 \sigma_3 \sigma_2 \rho_{32} +$

$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma^2_3 + 2 w_1 w_2 \sigma_1 \sigma_2 \rho_{12} + 2 w_1 w_3 \sigma_1 \sigma_3 \rho_{13} + 2 w_2 w_3 \sigma_2 \sigma_3 \rho_{23}$

Where σ(portfolio) equals the standard deviation (risk) of the portfolio, w_i equals the percentage of funds you invest in security i, σ_i^2 equals the variance of security i, and ρ_{ij} equals the correlation between security i and security j.

Some intuition here will help much. Notice what happens to w_i^2 in the first element on the right hand side of the equality sign as you add more and more securities to the portfolio. Because w_i is squared and because it is less than one, this term becomes small very quickly by adding more securities. Also notice that the variance term is small because we are squaring a number less than 1.0 (variance is a percentage). The multiplication of two small numbers, in turn, creates a product that is even smaller. My point is that the first element, $\sum w_i^2 \sigma_I^2$, quickly drops out of the equation as you add more and more securities to the portfolio. This element represents micro or diversifiable unsystematic risk within the portfolio. As a consequence, σ(portfolio) is not dependent on the variance (or standard deviation) of the individual securities except for a portfolio containing small numbers of securities (an undiversified portfolio). This means that σ(portfolio), the risk of the portfolio, depends on the correlation or interaction of securities within the portfolio, which is macro or undiversifiable systematic risk, opposed to absolute riskiness of the individual securities as measured by the variance (or standard deviation) of the security.

You need to understand this very important point because it is the cornerstone of modern portfolio theory. The key is diversification with a total portfolio perspective. Again, be sure you can recognize the relationship among the 3 models.

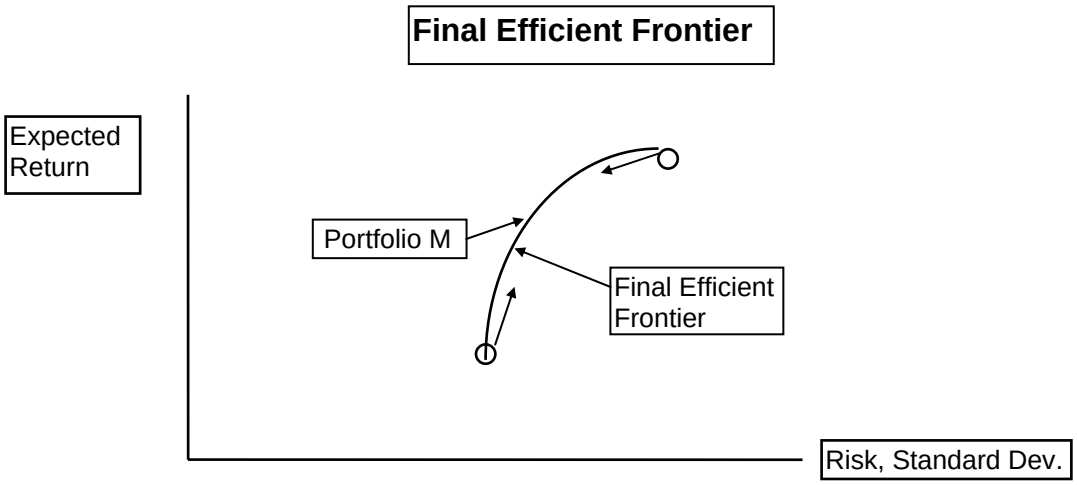
Let's now return to the final efficient frontier to answer a few important questions.

Question: How many securities lie between the 2 end points of the efficient frontier?

Answer: With your finger trace from the upper end of the final efficient frontier point, which contains only 1 security, down along the curve. As you move toward the center of the frontier, you are touching new portfolios where each one includes more and more securities. When you reach the center of the frontier (not exactly the center but close enough) the portfolio you are touching is the market portfolio, M. This is a very special portfolio since, as we discussed above, it is the only one that is perfectly diversified. If you started at the lower end point (MVP) and moved up, you would arrive at the center of the frontier. I wish to emphasize that the efficient frontier contains many portfolio but only one is the market portfolio, M, which is located approximately in the center of the curve.

Question: Where, on the final efficient frontier, is the one portfolio that contains all securities?

Answer: The portfolio approximately in the middle of the efficient frontier. This is a special portfolio called the market portfolio, M. It is the only portfolio on the efficient frontier that is perfectly diversified. In other words, as you work your way down from the highest end point or up from the lowest end point, you will eventually arrive at the midpoint, which is the approximate location of portfolio M (see graph below).



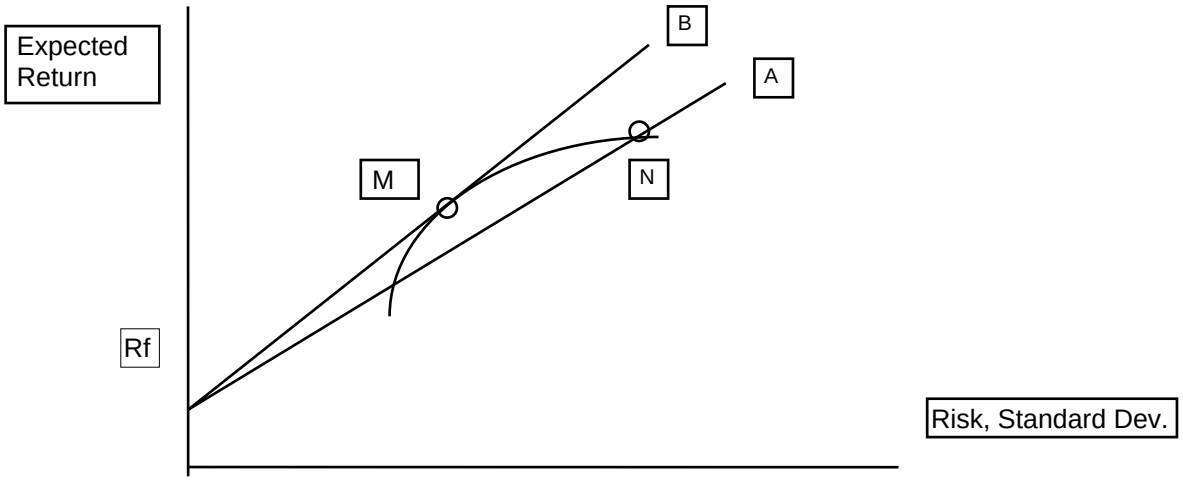
Ex post and ex ante efficient frontiers.
One last point before we leave the Markowitz frontier. Another potential item of confusion is the difference between the ex post and ex ante efficient frontiers. After-the-fact, you can construct an efficient frontier that precisely reflects historical returns, standard deviations, correlations. In other words, construction of an ex post efficient frontier is a rather straightforward exercise of graphing data.

Before-the-fact, however, construction of the efficient frontier is another matter. Most importantly, we cannot observe the necessary inputs. True, we can survey analysts regarding their expectations but even then, nobody has developed the perfect survey instrument. Additionally, the global market place makes surveys impractical. The bottom line is that construction of the ex ante efficient frontier is usually heavily dependent upon ex post data. Using ex post data to estimate ex ante data introduces estimation errors to the extent that the two differ. As I mentioned above, this is why the assumption of homogeneous expectations is so convenient, but also unrealistic. Reality is that every money management firm has its own view of what the ex ante efficient frontier looks like. I hope this discussion highlights the importance of economic forecasts and the skill involved in construction of the efficient frontier. It is not a mechanical process of just crunching a bunch of numbers.

This completes our development of the Markowitz (1954) efficient frontier, which shows all possible portfolios of risky securities excluding the risk-free rate. This is where Sharpe (1964) entered the picture and explored the question: What would happen to the Markowitz efficient

frontier if we add a riskless security (Rf) to the analysis? This is an important question because riskless securities exist in the market.

Let's use another graph to answer this question. Given the Markowitz efficient frontier, we can combine the risk-free rate with any number of risky portfolios on the Markowitz frontier. But remember that only one portfolio, the market portfolio, M, is perfectly diversified. Thus, combining the risk-free asset with non-diversified portfolios (for example, portfolio N to the right of portfolio M or, for that matter, any portfolio other than M) would result in a frontier with less efficient portfolios than a frontier generated from combining Rf with M. The graph below compares the efficiency of the new Sharpe frontier of line A (the combination of Rf with the non-diversified portfolio n) with a second new Sharpe frontier of line B (the combination of Rf with the diversified portfolio M).



Because frontier A is not as efficient as frontier B, you would rationally invest only in portfolios lying on frontier B. In other words, you want to invest in portfolios on the frontier with the highest slope, which is referred to as the Sharpe (S) measure. Again, be careful not to confuse the use of the ex ante S measure and the ex post S measure. We use the ex ante S measure to construct portfolios and in the asset allocation decision. We use the ex post S measure to evaluate performance. Later, when discussing portfolio management, we will address which particular portfolio you would select. You will see that this decision depends on your risk tolerance.

Capital Market Line

By the way, frontier B is called the Capital Market Line (CML). Bodie, Kane and Marcus call frontier A the Capital Allocation Line (CAL). Note that the CML applies only to portfolios, and not to individual stocks. Be careful not to confuse the CML with the Security Market Line, SML, which applies to both portfolios and individual stocks. The SML is the Capital Asset Pricing Model (CAPM) development of which is our ultimate goal for estimating k. Yet another line is the Characteristic Line, which is used to estimate beta. Just be careful in keeping these lines clear in your mind.

Question: Is there another efficient frontier with a slope greater than the slope of frontier A?

Answer: Yes and no, depending on whether you use ex post or ex ante data to construct the efficient frontier. The same discussion of the Markowitz frontier from the prior two pages applies here.

The main conclusion from the Sharpe model is that all investors will invest in some combination of Rf and M. For example, a risk averse investor would allocate all of his/her money in a 100%/0% (T-bill/M) combination, and a risk taker would allocation 0%/100% (T-bill/M) combination. In other words, Portfolio M plays a central role in the investment decisions of all investors.

Remember that measuring the risk premium in the Fisher equation is the reason we developed the Sharpe efficient frontier. Our goal is to develop an understanding the logic of the CAPM. We are now in a position to do this. The question now, given that all investors will invest some portion of their money in portfolio M, is: What is the relationship between any specific stock in M (i.e., General Motors) to the market portfolio M. I will give you two answers to this question: an ex ante answer and an ex post answer.

The ex ante answer is that we need to estimate what the future relationship between any stock and M will be in the future. Needless to say, this is very difficult if not impossible. Thus, most analysts make the simplifying assumption that the future will look like the past. Given this heroic assumption, we have to statistically measure returns for both the stock and an index, which is an estimate of M. Neither of these tasks are easy although AIMR performance presentation standards provide guidelines to follow such as use of total returns, use of time-weighted rates of returns, use of accrual accounting for dividends, and use of trade-date accounting.

Assuming we can overcome the problems associated with measuring returns, we still have the problem of selecting the proper index. A common assumption is that a broad-based index such as the S&P 500 is a good estimate of the stockmarket. Note that I said good, but not perfect by any means. Why? Because the market, by definition, contains all stocks. The S&P 500 contains only 500 stocks although it does cover a very large portion of the total market capitalization of all stocks traded on the New York Stock Exchange. Let's leave the question of whether the S&P 500 is a good index or not for another time. For now, let's accept it as a reasonable index and designate it as m.

From here the issues become somewhat tricky so be careful. We are interested in measuring the relationship or sensitivity of the returns of a specific stock (such as GM) to a market index, m. Because investors are forward-looking, they are not just interested in historical data except to the extent that the past would help project the future. In other words, you want to know how GM will react to future movements of m. Because analysts have no way of knowing the future for either GM or the index, they make the convenient assumption that the past contains important information about the future of both. Thus, they use historical returns on both GM and m to estimate the future relationship between the two. If this sounds like technical analysis, that's because it is!

The statistical tool that allows measurement of the returns on GM relative to m is beta. Remember that beta is a statistical estimate of relative risk since it measures the relationship between returns on an individual stock and returns on the market index, both of which are themselves estimates. Still, it is a nice tool because it has intuitive appeal for making investment decisions. Common definitions of beta are as follows:

Beta greater than 1.0	Aggressive Stock
Beta equal to 1.0	Average Risk Stock
Beta less than 1.0	Defensive Stock

Question: Can you use beta as a measure of risk for a stock in a 3-stock portfolio?

Answer: No, because a 3-stock portfolio is not diversified and beta is a measure of risk relative to the diversified portfolio m.

Now that we know how to calculate beta and what it means, let's see how to use it. Recall that our goal is to calculate the required rate of return, k, for an individual stock according to the Fisher equation, which is

k = risk-free rate + risk premium

Note that the risk premium is, by definition, the return in excess of the risk-free rate. Why? Because all securities in the market must provide the expectation of generating at least the risk-free rate. After-the-fact, this may or may not have happened but the expectation is that it will happen. No rational investor would invest in an investment without the expectation of achieving at least the risk-free rate. Thus, the risk premium is the expected reward for taking risk above the risk-free rate.

We saw that, according to Sharpe's CML model, all investors would invest some portion of their money in the market portfolio, as proxied by m. You can think of the expected return on m as an average--meaning that you would expect some stocks in m to generate a return above m and some below m. From this viewpoint, we can see that beta, as a measure of relative risk, distinguishes an average stock from all others. As previously stated, a beta greater than 1.0 (the average) signifies above average risk, a beta below 1.0 signifies below average risk, and a beta of 1.0 signifies average risk. Consequently, you may think that to calculate the risk premium on any stock you would simply multiply beta times the expected return on m, E(m). The problem

with this calculation is that E(m) already contains an expectation of reward in excess of Rf. At this point, I hope that you recognize that the risk premium for an individual stock, therefore, depends on the excess of E(m) above Rf, which you would then multiplied by beta in order to calculate the total risk premium for a specific stock. To get k, the required rate of return, you would add this risk premium to the risk-free rate.

It took a lot of tedious work to get this far, so we should do a quick review. Here are the major points we have covered in deriving an estimate of k:

1. the required rate of return, k, must provide an expected return greater that the risk-free rate due to the additional risk you take when investing in risky assets. According to the CAPM, the required rate of return on a stock depends on the security risk premium, which depends on the market risk premium. The difference between the security risk premium and the market risk premium is due to the security's beta, a measure of the stock's risk relative to that of the market.

2. selection of a security as the risk-free asset depends on your time horizon--for a short-term time horizon like 90 days, you should select a security that matures in 90 days; and for a long-term time horizon like 20 years, you should select a zero-coupon bond that matures in 20 years.

3. Sharpe's CML model provides a methodology for estimating the risk premium component of k. 2 major conclusions come out of the development of this model:

   a. all investors invest some portion of their money in the market portfolio, M, that is proxied by a market index, m, such as the S&P 500.
   b. beta is a measure of risk relative to the market; beta is useful for measuring a stock's risk premium only if the portfolio is well diversified.

Here is another time where you need to be careful. We know that k is referred to as the required rate of return. Unfortunately, many authors also refer to k as the expected rate of return. I am going to make an important distinction between the required and expected rates of return. I will continue referring to k as the required rate of return, and k' as the expected rate of return. This distinction is necessary in order to determine whether or not we believe a stock is accurately valued in the open market. We also need another term, alpha, that concisely conveys a buy, hold or sell signal. The following table shows these terms and their meanings.

Analysis	**Conclusion**	**Alpha (k' - k)**	**Action**
k' > k	stock is undervalued	positive	buy
k' = k	stock is fairly valued	zero	hold
k' < k	stock is overvalued	negative	sell

where:
k' = the expected rate of return
k = the required rate of return

We know how to calculate k, but do not know how to calculate k'. To do this, we need input from some other model such as a P/E model, which I believe is the easiest and one of the most widely used by analysts. Let's start with it and then see if we can use any other models to help us make the investment decision. I will illustrate how to conduct the analysis using U.S. Healthcare (USHC), the stock we looked at previously (see the enclosed Value Line data dated January 6, 1995). Let's calculate k' to cover the 1995 calendar year.

From the Value Line data, we see that the analyst (Vik Malhotra) projects USHC to achieve a P/E of 18 during the 1997-1999 time frame. If you were to view this projection relative to the past 15 years, a P/E of 18 appears reasonable. Let's use it as a projection of the P/E ratio for the end of 1995. If, perhaps, you did believe this projection, you could change it to whatever you believe is most likely.

If we further believe that Malhotra's expectation that USHC will earn $2.85 per share in 1995, you would multiply 18 times $2.85 to get a projected price of $51.30 per share by the end of 1995. Again, you could adjust the earnings estimate according to your expectations if you do not agree with Malhotra. Using the projected price of $51.30, you would expect to receive a total return on this stock of % calculated as follows:

k' = [(projected price - current price) / current price] + (expected dividend / current price)
= ($51.30 - 40) / 40 + $.88 / 40

= 28.25% + 2.2%

= 30.45%

The current price of USHC is located at the top of the Value Line sheet, and the 1995 projected dividend of $.88 is located 4 lines down from "Revenues per share."

Next, we need to calculate the required rate of return (k) as follows:

k = Rf + beta [E(m) - Rf]

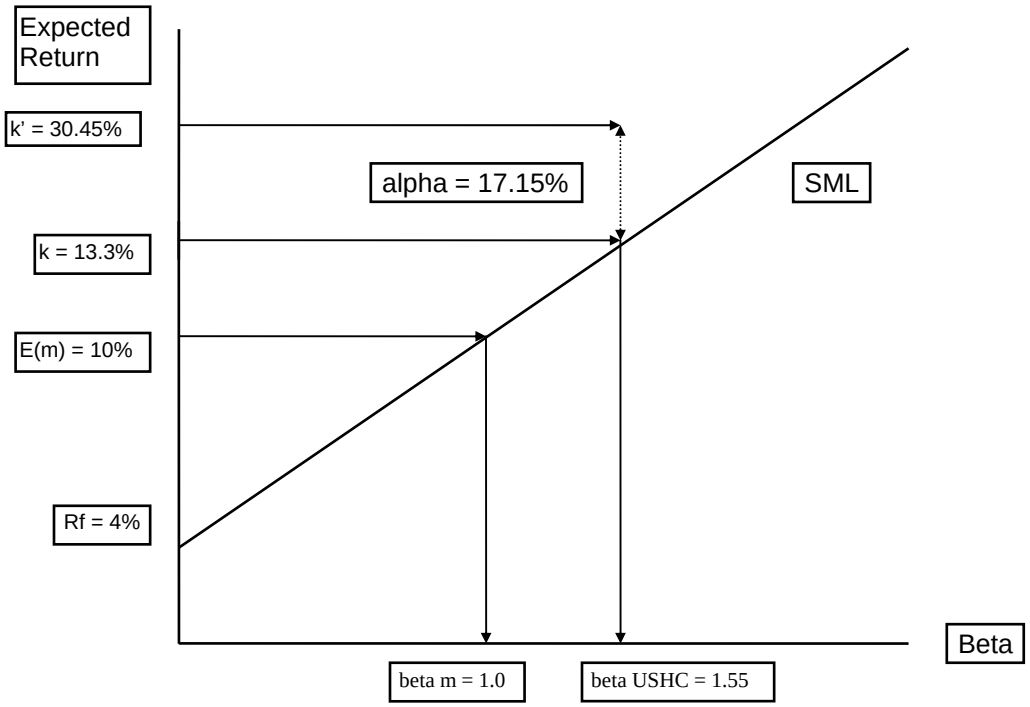
Assuming an annual risk-free rate of .04, an E(m) return of 10% (we will see later one method of estimating E(m) by using historical data) and Value Line's estimate of USHC's beta of 1.55, we would estimate k as follows:

k = .04 + 1.55 (.10 - .04)
= 13.3% (annual)

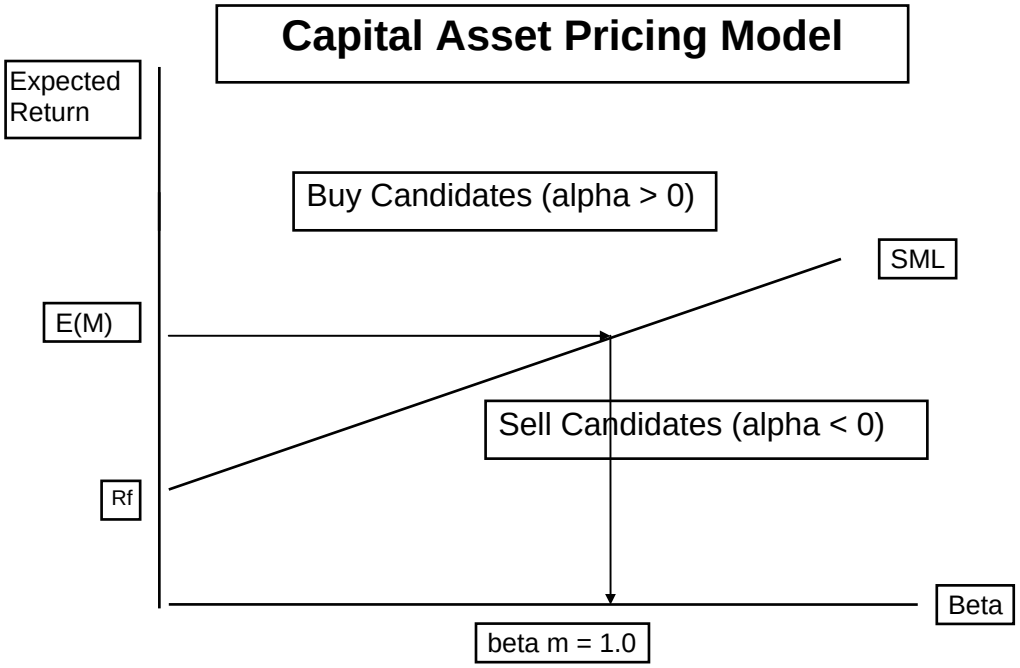
Given these two inputs, we can calculate alpha (= k' - k) as follows:

alpha = 30.45% - 13.3%
= 17.15%

Because alpha is positive, the conclusion is that USHC is a buy candidate--a very strong buy signal, at least according to this model. The graph below illustrates this analysis.



From this analysis, you can clearly see why the CAPM, as reflected in the Security Market Line (SML) is called a pricing model. All stocks with expected returns (k') that plot above the line (alpha > 0) signal buy candidates, and all stocks with expected returns (k') that plot below the line (alpha < 0) signal sell candidates. Another way of saying this is that if capital markets are efficient, the required rate of return (k) will equal the expected rate of return (k') and alpha will equal zero. Similarly, if capital markets are inefficient, k will not equal k' meaning investors can find positive alphas.



Capital Asset Pricing Model

A few comments about the CAPM. First, the CAPM is a product of Sharpe's CML model because it builds on the main conclusion of the CML that all investors will always invest some portion of their money in the market portfolio, M. This conclusion allows construction of a new efficient frontier (a straight line, which is the CML) by estimating only 3 variables: Rf, E(M), and the standard deviation of M. Recall that you would need a huge number of estimates to generate the Markowitz efficient frontier (the curve). You would need expected returns and standard deviations for every stock in the universe, plus covariances for every possible 2-security combination in the market. This impracticality motivated Sharpe to develop his model.

Note: The CAPM is the SML, not the CML. Many people get this confused. Just remember that the SML comes from the CML since the main conclusion of the CML is the importance of M. The CML is a portfolio model; the SML is mainly an individual stock model although you could think of a portfolio as an individual stock. Given your estimates of Rf and E(M), you can then construct the SML since you know that the beta of M is 1.0. Once you have the SML, you need to estimate beta for the stock you are analyzing. This estimate comes from the characteristic line, which is a simple regression model of expected returns for the stock relative to the expected returns on M. Notice that I said expected returns, not actual returns. Still, many people use actual historical returns as estimates of expected returns. Given beta for the stock, you can then calculate alpha to determine whether to buy, hold or sell the stock.

Second, the CAPM is a single-factor pricing model where the market portfolio is that one source of systematic (undiversifiable) risk. That is, the market portfolio influences the returns of all stocks through beta, which measures the relationship between the returns on any specific stock and the returns on the market portfolio. Due to this reliance on only one factor and the great

supply of empirical evidence that does not support the model, Ross developed the Arbitrage Pricing Theory (APT) that considers the influence of many factors on the pricing a security. Thus, APT is a multifactor model whereas CAPM is a single factor model. I will have more to say about APT below.

Question: What is the definition of alpha?

Answer: Alpha is a risk-adjusted return in excess of the required return. It is risk-adjusted because beta adjusts for risk in the required rate of return and the value of alpha depends this required rate of return.

Question: Can you expect to beat the market if it is efficient?

Answer: This is a trick! The answer depends on the meaning of "beat the market." If it means achieving an absolute return greater than some market index, then, yes, you would expect to beat the market simply by investing in high beta stocks. If, on the other hand, it means achieving a risk-adjusted return greater than some market index, then, no, you cannot beat the market.

This last question raises another interesting and important question. If the CAPM assumes that the market is efficient, how can alpha ever equal anything but zero? To address this question, we need a clear definition of market efficiency.

Market efficiency--is concerned with how quickly security prices adjust to new information. If the market is perfectly efficient, stock prices adjust immediately. If the market is not efficient, stock prices adjust slowly.

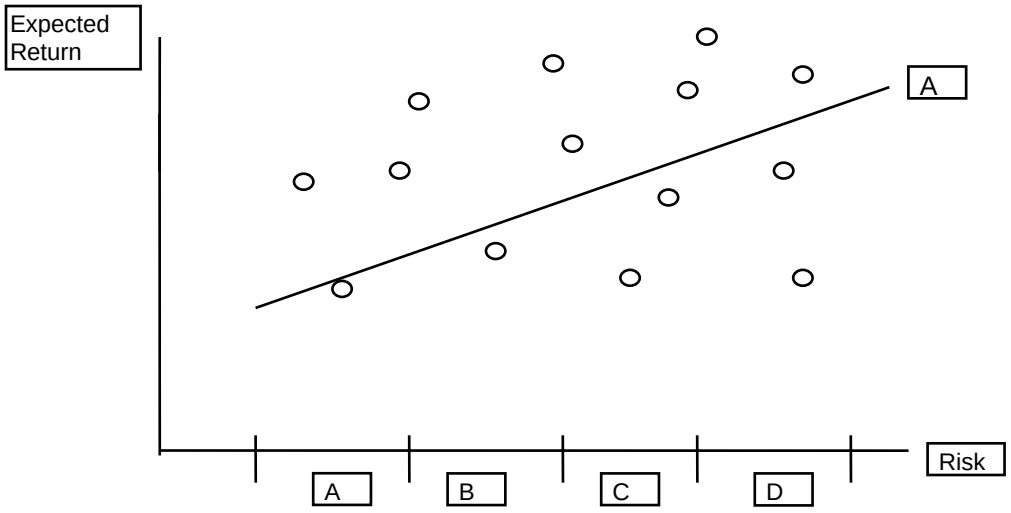
Non-zero alpha values can exist in a highly efficient market, but not in a perfectly efficient market. In a highly efficient market, non-zero alphas would disappear depending on the degree of efficiency; that is, how quickly information flows throughout the marketplace. As you can see, market efficiency is a relative concept stated in terms of 100% efficient and less than 100% efficient. Personally, I would place market efficiency around 85% for large, well known stocks like (IBM) and progressively lower as a particular stock is less known. Some analysts refer to less known stocks as neglected meaning that few analysts follow the stock. Typically, neglected stocks are small-caps. The implication is that the market prices small-cap stocks less efficiently than large-cap stocks. The key to understanding market efficiency is understanding the speed with which information flows from investor to investor.

Problems with the DDM

The DDM is suspect for estimating the intrinsic value of a single stock because of problems with estimating k and g. I have focused much attention on the DDM in these notes because such a discussion allows me to discuss many important and related topics. Still, I suggest that you use the DDM with caution for estimating the intrinsic value of a single stock. You may, however, feel more comfort when using it in estimating the intrinsic value of the market using, for example, data on the S&P 500. For an index that contains many stocks, estimation errors tend to offset each other and the model is much more reliable. Even then, you should use the DDM as only one of your many tools.

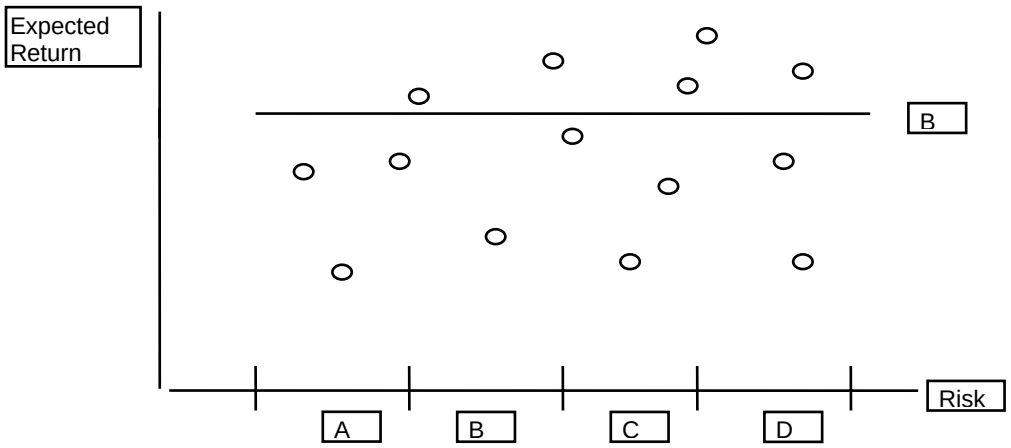
A Stock-picking Model. Before we leave the CAPM and alpha, let's see how you might use these concepts if you wanted to construct a portfolio of carefully selected stocks. To set the stage, let's assume that you believe the market is something less than perfectly efficient (otherwise, you would simply invest in a mutual fund that mimics the movement of the market, an index fund) and you have superior analytical skills. What I will now describe is referred to as the bottom up approach to picking stocks, which usually requires a computer data base.

Let's say that you have 1700 stocks in your data base. Taking advantages of the wonders of modern technology, you first calculate expected return estimates (k') for all 1700 stocks and categorize each k' according to a risk grouping defined from A (low risk) to D (high risk). You then plot all your k' estimates on a graph as shown on the scatter diagram below. Next, you draw a line through the data that divides the data according to your desired level of diversification. For example, you could draw the line such that it separates the top 10% of the stocks from the bottom 90%. Alternatively, you could separate the top 20% from the bottom 80%, and so forth. The larger your top grouping, the greater the number of stocks you will capture as buy candidates. This top grouping, then, represents a portfolio of stocks that you believe will outperform, on a relative basis, all the other stocks in your universe.



Question: Is the line you drew through the data set I just described the SML?

Answer: No, it is not. It is simply a line showing your style of investing. The line could slope upward and cut the data in half, which implies that you wish to construct a portfolio comprised of stocks in all 4 risk categories (see Line A above). Alternatively, you could draw the line flat in such a way that you capture only high-risk securities (see Line B below). In other words, the line you drew represents the tradeoff between your return expectations for each stock and a category of risk. The SML represents the tradeoff between your required returns and beta, which is a statistically defined measure of risk. The SML intersects two points on such a graph: (1) the risk-free rate, and (2) the point where E(m) and beta of 1.0 intersect.



Arbitrage Pricing Theory (APT)

You need to understand two key points in order to understand APT

1. The CAPM is a special case of APT
2. APT is a multifactor model whereas CAPM is a one factor model (the market portfolio is the one factor)
3. According to APT, the return on any security equals an expected return and an unexpected return.

Let's begin with #3 and state it in equation form as follows:

Ri = Expected return + Unexpected return

According to Chen, Roll and Ross, the unexpected return is driven by 4 macro factors: (1) unexpected changes in the level of production in the economy, (2) unexpected changes in inflationary expectations, (3) unexpected changes the spread between the short and long end of the default free yield curve, and (4) unexpected changes in the spread between low and high default rates on risky debt. The four-index market model looks like this:

$$R_i = a_i + b_{i1} F_1 + b_{i2} F_2 + b_{i3} F_3 + b_{i4} F_4 + e_i$$

where

a_i = security i's Expected Return
b_{ik} = the ith security's return responsiveness to factor k (k = 1.....4)
F_k = the return on the non-diversifiable factor k
e_i = the residual term.

In equilibrium, returns are perfectly expected (assuming homogeneous expectations in a perfectly efficient market) meaning that all the b_{ik}'s equal zero--nothing is unexpected. The implication is that for an investment with no risk (zero b_{ik}'s), the unexpected return to a security exposed to these F factors should equal zero and a portfolio comprised of these securities should, likewise, achieve no unexpected return. If the market is not in equilibrium, then arbitrage will occur meaning that an investor can achieve a positive return without any investment and without any risk. The investor could do this by shorting/selling the overvalued securities and using the proceeds to buy undervalued securities.

In equilibrium, if you invest nothing you should not get anything in return and arbitrage should not occur. Consequently, the additional return to a diversified portfolio with no new investment of money is zero (no opportunity to short overvalued securities). Under the condition of equilibrium, the expected return on any security is then

$$\text{Expected Return} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \lambda_3 b_{i3} + \lambda_4 b_{i4}$$

where λ_0 equals the risk-free rate, and λ_i (i = 1,...4) equals the risk premium for the ith factor. If there is only one factor and that factor is the market portfolio, then the above equation looks like this:

$$\text{Expected Return} = Rf + \lambda_1 b_{i1}$$

where $\lambda_1 = E(Rm) - Rf$ = the risk premium for investing in the market portfolio.

Before leaving APT, let's take a look at how arbitrage works. Arbitrage opportunities allow you to generate a positive return without any risk or without putting up any additional cash. For example, gold is selling at $400 per ounce in Zurich and $380 per ounce in New York. In this instance, you would buy gold in New York and sell it in Zurich until your actions and the actions of other arbitrageurs force the prices in each market together. The arbitrage requirement means you must be able to take both long and short positions in the same security where revenues from short positions and exactly offset costs from long positions. In other words, arbitrage means no risk. As stated above, if investors eliminate all arbitrage opportunities through selling (shorting) and buying as in the gold example, the expected return from an arbitrage portfolio is zero.

Relationship between CAPM and APT

Comparison of the SML Line with the 1 Factor APT

In both cases there is a simple linear relationship between expected excess returns and a security's beta. The difference between the two models lies in the interpretation of the factor. According to the CAPM, this is the market index M (the set of all risky securities). According to APT, the factor could be the market portfolio, but not necessarily. A problem for APT is that the model does not identify the factors. Chen, Roll and Ross gave us their determination of the 4 factors, but they did not arrive at this determination based on the theory. CAPM, however, identifies the market portfolio as the one factor.

Real Estate Investment Analysis

In the discussion that follows, I will focus on two different models for estimating the value of a real estate investment, the similarities and dissimilarities of investing in real estate and bonds, and the diversification benefits of real estate especially during periods of inflation. Throughout the discussion, I will address the necessary assumptions you must make when using each model along with potential problems. This discussion is particularly important for understanding the factors affecting value and how real estate fits in the asset allocation decision. I will begin by showing that real estate valuation is highly related to stock valuation in that they both use the same basic theoretical model.

The cap rate approach to valuing real estate
According to the dividend discount model (DDM), the intrinsic value of a stock (Po) equals the present value of all future dividends discounted at the stockholder's required rate of return (k) and growing at a constant rate (g). Mathematically, the model is:

$$Po = D_1 / (k - g) \quad (5)$$

The cap rate approach to real estate valuation looks very similar to the DDM and is stated as:

$$V = NOI / \text{capitalization rate} \quad (6)$$

where:
V = intrinsic value of the investment
NOI = net operating income

capitalization (cap) rate = the discount rate *(Note: Be very careful with the term "cap" rate and "discount" rate. Many people, including AIMR, use these terms interchangeably although I wish they did not. The problem is that the term "discount" rate is most often used with the letter k. From the above model, the cap rate equals k - g making k equal to the cap rate plus the growth.*

NOI is similar to D1 and the cap rate is similar to k - g in the dividend discount model. When using this, you need two estimates: (1) projected NOI for the next period, and (2) an estimate of the cap rate, which usually comes from comparable properties (properties that are as similar as possible and that sold very recently). If, for example, you wish to estimate the value of an investment with a projected NOI of $10,000 for next year, you would like to find 3 comparable properties and observe their recent selling prices and NOIs. Suppose you obtain data on 3 recently sold properties with the following net operating incomes and sales prices:

	Property A	**Property B**	**Property C**
NOI	$9,500	$11,000	$8,500
Sales Price	$90,000	$104,762	$89,474
Cap Rate	.1055	.105	.095

The average of the 3 cap rate is 10.2% (rounded). Using the cap rate approach to valuation, you would estimate the value of the property at $98,039 ($10,000 / .102). The assumption you are making is that the NOI will grow at a constant rate forever, although you do not know what that growth rate is from the information you have. By definition, you could subtract the cap rate from k (if you knew k) to get g.

A cash flow model that includes projected after tax cash flows over an assumed 3-year holding period is presented below. I will refer back to this model in the discussion that follows it. Note the calculation of NOI, after tax cash flows (ATCF) for each of the 3 years, and after tax net proceeds from sale (ATNPS) . We will use ATCF and ATNPS when determining the value of the property using the discounted cash flow approach to valuation.

CASH FLOW MODEL

	Year 1	Year 2	Year 3
Effective Gross Income (Rents)	$36,288	$38,827	$41,544

- Operating Expenses	$13,466	$14,408	$15,417
Net Operating Income (NOI)	$22,822	$24,419	$26,128
- Debt Service (Mortgage Payment)	$21,280	$21,280	$21,280
Before-Tax Cash Flow (BTCF)	$1,542	$3,139	$4,848
- Taxes (Savings)	($1,140)	($668)	($158)
After-Tax Cash Flow (ATCF)	$2,682	$3,807	$5,005
NOI	$22,822	$24,419	$26,128
- Interest on Debt (from amortization schedule	$20,350	$20,259	$20,146
- Depreciation	$6,545	$6,545	$6,545
Taxable Income	($4,073)	($2,385)	($563)
x Marginal Tax Rate	0.28	0.28	0.28
Taxes (Savings)	($1,140)	($668)	($158)
Forecasted Selling Price			$283,618
- Selling Expenses			$19,853
- Unpaid Mortgage Balance (from amortization schedule)			$181,915
Net Proceeds Before-Taxes			$81,850
- Taxes (@ .28)			$10,752
After-Tax Net Proceeds from Sale (ATNPS)			$71,098
Forecasted Selling Price			$283,618
- Selling Expenses			$19,853
- Book Value (= Purchase Price - Accumulated Depreciation)			$225,365
Gain on Sale			$38,400
x Tax Rate			0.28
Taxes Due on Sale			$10,752

Notes:

1. Purchase Price (t = 0) = $245,000
2. Equity (down payment) = $60,000
3. Loan (Mortgage) = $185,000 on terms of 11% compounded annually for 30 years
4. Appreciation of Property = 5% per year for 3 years (holding period)
5. Depreciation = Straight Line over 27.5 years (Land = $65,000, Building = $180,000)--only building is depreciable.
6. Selling Expenses = 7% of Selling Price
7. Growth Rate of Effective Gross Income = 7% per year
8. Operating Expenses = 37% of Effective Gross Income

Now before we see how to use the discounted cash flow model, let's look at problems in estimating value using the cap rate approach and discuss each problem separately. The cap rate approach:

1. ignores leverage
2. ignores taxes
3. ignores the terminal value of the property
4. assumes that comparables are really comparable

The cap rate approach ignores leverage. Remember that we discussed M&M's theory (no taxes), which says that the value of a firm is independent of the manner in which it is financed. If this is true, leverage should not impact the price you are willing to pay for a property. There is no question that greater leverage can lead to a greater internal rate of return (IRR). But greater leverage also leads to greater risk meaning that the required rate of return is higher. M&M would argue that the higher risk offsets the greater potential return such that the value of the property is the same for the investor who will use high leverage and a second investor who will not use any leverage. My point is that while the cap rate approach ignores leverage, the M&M argument states that this is not a problem.

The cap rate approach ignores taxes. What about taxes? In other words, would two different investors, one in the 40% bracket and the other in the 15% bracket, be willing to pay the same price for the same property? After all, depreciation and interest expense deductions are more valuable to the higher bracket investor. It appears that the investor's tax bracket is an important investment consideration for real estate assuming that both investors evaluate the investment as having the same degree of risk. This follows the M&M argument that taxes encourage the use of as much leverage as possible. If so, then sellers would have more incentive to sell to higher bracket investors than to lower bracket investors. This sounds good, but I doubt it actually happens in the market place. Actually, the same issue is debated in the stock market. At this time, I do not believe that the tax issue is settled.

The cap rate approach ignores the terminal value of the property. Next, what about the terminal value? The cap rate approach assumes an infinite stream of future cash flows with no terminal value. This is obviously not true. Still, you could reinvest the tax savings from the depreciation deduction back into the property in order to maintain the physical attractiveness of the property. Doing this would enable you to generate a cash flow stream to the investment for a very long period of time, although not forever since depreciation eventually ends. So, on one hand, it appears that the cap rate approach ignores a potentially important source of cash flow by ignoring terminal value of the property. On the other hand, the terminal value may be so far into the future that its present value is relatively unimportant. Thus, it looks like we have not settled this issue either.

The cap rate approach assumes comparable properties are comparable. Finally, what about the problem of finding comparable sales? This is a practical problem because most real estate properties are unique. Not only do you have to find comparables with similar physical characteristics to the subject property, but they should have been sold very recently, like yesterday. Needless to say, finding comparables is not an easy task.

In the final analysis, the cap rate approach is a quick and easy method of determining the approximate value of the investment. In other words, it is a place to begin your negotiations. Its main drawbacks are that it ignores taxes, it ignores the terminal value of the property, and it assumes that comparable properties are, indeed, comparable.

Discounted cash flow model to valuation

Let's now see how to use the discounted cash flow analysis calculates value. This approach is based on calculating the present value of future cash flows that you would reasonable expect to receive over the 3-year holding period. You can look at this approach from two different

perspective: (1) either as a rate of return called the internal rate of return (IRR), or (2) as a net present value (NPV). The IRR is the rate of return you expect to receive from investing $60,000, your equity (down payment), today with the expectation of receiving After Tax Cash Flows (ATCFs) from operations over the next 3 years plus the After Tax Net Proceeds from Sale (ATNPS) at the end of 3 years. The NPV is the difference between the present value of benefits and the present value of the costs.

Internal Rate of Return (IRR)
The example shows that by investing $60,000 today, you expect to receive the following after tax cash flows:

Cash Flow Analysis

	Year 0	Year 1	Year 2	Year 3
ATCF	-$60,000	$2,682	$3,807	$5,005
ATNPS				$71,098
Total	$60,000	$2,682	$3,807	$76,103

Internal Rate of Return (IRR) = 12%
Net Present Value (NPV) = $2,511 (at Required Rate of Return = 10%)

These after tax cash flows generate an IRR of 12%. Because the IRR exceeds the required rate of return of 10%, the project is acceptable. From the other perspective, these cash flows generate an NPV of $2,511 (at a discount rate of 10%). Because the NPV is greater than 0, the project is also acceptable from this perspective. In this case, both criterion provide the same conclusion. In some instances, the two criteria provide conflicting results. If this happens, the NPV is the more theoretically correct since it says that you expect to increase your net worth by $2,511 by investing in this property. Still, most investors find it easier to relate to a rate of return calculation.

Given the above expectations including a purchase price of $245,000 of which you put down $60,000 and borrow the remaining $185,000, you would have an attractive investment assuming your projections are accurate. Actually, both the IRR and NPV suggest that you could pay more than $245,000. At this price, you expect to receive 2 percent more than you require (IRR of 12% minus the required return of 10%). This 2 percent excess return, which is risk adjusted, is similar to the alpha that equity analysts continually strive to achieve. This is a good time to discuss the efficiency of each market.

The academic literature generally does not consider the real estate market as efficient as the stock market. An important reason for this is because real estate is mostly a local market whereas the stock market is more national and international. Information flows easier in the stock market than the real estate market. Additionally, inside information in the real estate market is not illegal, but it is in the stock market.

In this example, you can see that the cap rate approach gives an estimated value of $228,220 ($22,822 / .10) assuming a cap rate of 10%. The discounted cash flow analysis, however, shows that at a purchase price of $245,000, you expect the investment to generate an internal rate of return of 12% and a net present value of $2,511 (at a discount rate of 10%). In other words, you

could purchase the property for $247,511 ($245,000 + $2,511) and still receive your required rate of return on 10% assuming your projections are accurate. In reality, you would probably begin negotiations at a price of around $228,000 and, depending on the level of confidence you have in your projections, be willing to pay as much as $247,511. Remember that the discounted cash flow model assumes a 3-year holding period. You would want to analyze the investment under varying holding-period assumptions.

If the investor in our example were the management of a corporation making a capital budgeting decision and decided to make the investment, the stockholders may or may not agree with this decision. If the stockholders do agree, the price of the stock should increase by the amount of the net present value. On the other hand, the stockholders may not agree either because they reject management's projections or determine that the property has more risk than management surmises. A necessary assumption is that the stockholders have sufficient information with which to analyze the investment. Usually, this is not the case although the security analysts usually do and it is the analysts that exert considerable influence on the stockholders.

Decomposition of rate of return

We can decompose the rate of return to a real estate investment similar to how we decompose the return on a stock. That is, the total return to a real estate investment has a yield component (BTCF / Equity) and a capital gains or appreciation component (ending value - beginning value / beginning value). Additionally, real estate has a tax shelter component (tax saving / equity) and an equity component (principal buildup resulting from payoff of the mortgage loan). The following table shows the relative importance of each component:

Components of Total Return for a Real Estate Investment

Component	Relative Importance
Cash Flow (yield)	Depends on amount of leverage--higher leverage means lower cash flow, and vice versa
Capital Gains (growth)	Affected by inflation and supply/demand factors--historically, 1-3% greater than inflation, holding supply/demand constant
Tax Shelter	Depends on length of depreciation schedule and tax rates--tax shelter becomes more important as the depreciation schedule shortens and tax rates increase, and vice versa
Equity Buildup	Slow in initial years of loan amortization and fast in later years

Similarity of investing in real estate and investing in bonds

In addition to the similarity between analyzing real estate and stocks, you can also see a distinct similarity between real estate and bonds. Each has an initial investment (at t=0); each has periodic cash flow (although the bond is fixed); and each has a terminal value (again, the bond is fixed). You calculate the 2 rates of return using the same discounted cash flow methodology:

Investment	Rate of Return

Bond	Yield to Maturity (YTM)
Real Estate	Internal Rate of Return (IRR)

The most important difference between investing in a bond and investing in real estate is the ability of the real estate investment to pass through inflation to the lessee in the form of higher rents. Additionally, real estate prices (similar to the maturity value of a bond) usually appreciate as inflation increases due to the higher costs of building comparable structures. In other words, you have the ability with real estate to adjust the cash flows (ATCFs and ATNPS) as inflation increases. With a bond, you do not have this ability since its cash flows (coupon payments and maturity values) are fixed. This is why real estate is considered an inflation hedge while bonds are not.

I have directed my comments to the income approach to valuing income producing real estate. Additionally, the real estate appraisal profession has developed two other methods: the cost method and the comparable sale method. The following CFA question addresses these two other methods as well as several of the points I have made concerning the income approach

Duration of real estate vs. duration of a bond
With a bond, cash flows (coupon payments in the numerator of the bond model) are fixed. Higher inflation leads to a higher yield to maturity (in the denominator of the bond model) and the price of the bond decreases. The reason this inverse relationship occurs is because the present value of the cash flows declines as the discount rate (yield to maturity) increases. Recall that according to the Fisher equation, inflation drives the discount rate.

With real estate, however, cash flows (in the numerator of the cash flow model--see above example) are not fixed. Higher inflation leads to a higher internal rate of return (in the denominator of the discounted cash flow model) and you might suspect that the value of the property would decline just like with a bond. But remember that real estate cash flows are not fixed, and that the higher inflation should result in higher rent levels that, in turn, lead to higher after-tax cash flows assuming that rents increase faster than expenses. Even if rents and expenses increase by the same percentage, NOI will increase since operating expenses represent only a percentage of the rents. This is an important point, and I would like to illustrate it with an easy example as follows:

	Year 1	**Year 2**
Rent	$100	$110
Operating Expenses	50	55
NOI	$50	$55

In this example, inflation increased by 10% between years 1 and 2 and is reflected in both higher rents and higher operating expenses. NOI also increased by 10% since rents exceed operating expenses. Thus, landlords can pass through higher inflation to the renters in the form of higher rents meaning that higher inflation actually helps landlords with higher NOI. Additionally,

inflation helps landlords with higher BTCFs since debt service (mortgage payment) is constant even though taxes are increasing but not enough to offset the higher BTCF.

My point is that ATCF's increase with inflation to the extent that the landlord can pass through the inflation to the renters. Be careful, however, because landlords cannot automatically do this. Local supply and demand factors affect the landlord's ability. Specifically, if the local market is oversupplied with rental space, this factor may prohibit the landlord increasing rent. The level of competition in the market is another important factor. So much for the impact of inflation on ATCF. The next question is, how does inflation impact the terminal value of the property?

In the case of a bond, the maturity value of the bond is fixed. For real estate, however, this is not the case. Remember that an important factor affecting the value of any asset is the cost to replacing that asset. Building costs (lumber, brick, cement, etc.) at the time of termination determine the replacement value of the structure. The land value will also reflect inflation for the same reason. Thus, the seller will pass through inflation to the buyer at the time of termination due to higher replacement costs. In our above example, we projected the selling price three years into the future to increase by 5 percent per year largely because of inflation.

In summary, the following table show the 2 types of cash flows to both a bond and a real estate investment and how inflation affects each:

	Cash Flows During Holding Period	**Cash Flow at Termination**
Bond	Coupon payments (fixed)	Maturity Value (fixed)
Real Estate	ATCF's (not fixed)	ATNPS (not fixed)

Duration

With fixed cash flows for a bond, calculating duration is a straight forward mathematical matter. With real estate, however, calculating duration is not so easy. In the discussion that follows, I use the term "duration" to mean how long it takes to receive back you initial investment. This is an intuitive (non-mathematical) meaning. For example, a 90-day T-bill has a duration of 90 days, its maturity. You get your money back at the end of 90 days. A 20-year zero-coupon bond has a duration of 20 years. As the coupon rate of a bond increases, duration decreases because the coupon payments mean you get your money back quicker.

A related meaning is how sensitive the value of an asset is to changes in interest rates. The shorter the duration, the less sensitive the asset is to changes in interest rates; and the longer the duration, the more sensitive the asset is to changes in interest rates. Using either meaning, and they are equivalent, you would want to invest in short duration investments during periods of high inflation in order to take advantage of rolling over your investments into higher yielding vehicles. Alternatively, you would want to invest in long-duration bonds during periods of declining inflation.

Impact of inflation on both real estate and bonds

The concept of duration equally applies to real estate. During periods of high inflation, you want to invest in real estate investments with short durations. A good example of this is a hotel, where room rates may virtually change daily (duration of lease is one day). On the other hand, during periods of declining inflation, you would want to invest in projects with long-durations. An example of this is a shopping center or office building where lease rates may not change for several years (duration as long as term of lease). The crucial point is how often you renegotiate the lease. The longer the term of the lease, the longer the duration, and vice versa. This is why lessors (owners of the real estate) want to renegotiate leases more quickly during inflationary times than when inflation is not a treat. My point is that the shorter the duration of the real estate investment, the easier it is to pass through inflation to the renter.

We now have a good idea of how inflation affects the cash flows of a bond and an investment in real estate differently. Recall that the second definition of duration is a measure of how sensitive the value of an asset is to changes in interest rates. Since cash flows accruing to a bond are fixed, the present value of a bond is adversely affected by inflation; and the longer the duration of the bond, the worse the impact. On the other hand, the present value of real estate is not necessarily adversely affected by inflation. Quite the contrary: inflation usually enhances the value of real estate. In other words, real estate acts as an inflation hedge whereas bonds do not.

Correlation analysis

Because inflation affects the two investments in different ways, you would expect to observe different correlation coefficients (R) between them. Specifically, you would expect to see low or even negative correlation between bonds and real estate ($R_{RE/B}$). Likewise, you would expect to see a positive correlation between real estate and inflation ($R_{RE/I}$), and a negative correlation between bonds and inflation ($R_{B/I}$). Not surprisingly, this is what Ibbotson and Siegel found over the period from 1971 to 1987. The following table shows these correlations:

	LT Govt. Bonds	**Real Estate**	**Inflation**
LT Govt. Bonds	1.0		
Real Estate	-.31	1.0	
Inflation	-.59	.50	1.0

Source: Ibbotson and Seigel, "How to Forecast Long-Run Asset Returns," Investment Management Review, now Investing Magazine, as reported in Appendix Table B of Cases in Portfolio Management, AIMR, 1990.

These correlations show that real estate has good diversification benefits. That is, real estate returns tend to increase as inflation increases due to the positive relationship ($R_{E/I}$ = .50), and increase when bond returns decrease due to the negative relationship ($R_{E/B}$ = -.31). Bonds, on the other hand, do not perform well during inflationary times due to its negative relationship ($R_{BI/I}$ = -.59). These data strongly suggest that during inflationary time, a portfolio including real estate is, at least, somewhat protected. The greater the protection you want, the more real estate you would have in your portfolio. This is the good news. The bad news is that over long periods of time, real estate tends to underperform stocks. Ibbotson and Siege report the following total return data for the period from 1947 to 1982:

	1947 - 1982
Stocks	12.4%
Real Estate	8.3%
Bonds	4.1%

Source: Ibbotson and Seagull, "Real Estate Returns: A Comparison with Other Investments," Journal of the American Real Estate and Urban Economics Association, Fall 1984:

On the other hand, in deflationary times a portfolio including real estate does not perform so well. The obvious question is, do you know beforehand when inflation will be a problem? This requires superior forecasting ability that most investors do not possess. As a consequence, diversifying your portfolio ahead of time is a wise strategy. An alternative strategy is to focus on stocks and not worry about inflation since stocks usually outperform both real estate and bonds over long periods of time. Over the period from 1947 to 1982

Be careful with these data because real estate returns are based on appraised values, which cause smoothing and possible inaccuracies. Generally, however, other studies covering long periods of time show similar results. Over any short period, the results and conclusions may differ significantly. More on the diversification benefits of real estate in the portfolio management notes. In terms of expected return, you would expect real estate to generate returns below stocks but higher than bonds.

Investing in Ginnie Mae Certificates and Collateralized Mortgage Obligations

I. Introduction
Due to low interest rates on money market instruments, many people have questions about the higher yields available with Government National Mortgage Association (Ginnie Mae) pass-through certificates and Collateralized Mortgage Obligations (CMOs). In addition to offering attractive yields to investors, these two instruments represent an important means of providing liquidity to the housing market. A national market in mortgages allows capital to flow into the housing sector from any region of the country independent of local credit conditions.

The purpose of article is to explain the basics of investing in Ginnie Maes and CMOs without getting too bogged down in detail. I will focus more on Ginnie Maes than CMOs because Ginnie Maes provide the underlying cash flows to CMOs. Keep in mind that the concepts I discuss apply equally to agency pass-throughs (Fannie Maes and Freddie Macs) as well as to conventional pass-throughs (issued by thrifts and commercial banks). I will use Ginnie Maes for illustration purposes.

II. Ginnie Maes
Overview. Ginnie Mae pass-through certificates represent an investment in a pool of homeowner mortgages. The pool is comprised of mortgages created by a lending institution such as a local Savings and Loan who then, with the help of an investment banker, sells portions of the pool to investors like you and me. This process is called mortgage *securitization*, meaning that you can invest in a piece of John Doe's mortgage without directly lending him any money. As John Doe

and other homeowners repay their mortgages each month to the lending institution, the institution passes through the mortgage payment (principal and interest less a servicing fee) to you and the other investors in the pool.

Prepayment Risk. **The key to understanding Ginnie Mae investments is prepayment risk**. The problem is that you do not know when a homeowner will prepay his/her mortgage. Prepayment occurs when homeowners refinance their mortgages due to declining interest rates or when they move. As a consequence, you do not know the maturity of your Ginnie Mae at the time of purchase. Prepayment is important because it represents a return of principal that, in turn, means you have less money on which to earn interest. Prepayment is a problem because as you receive your principal back, you are forced to reinvest the principal at lower interest rates that initiated the prepayment in the first place.

Since prepayment is an important factor affecting the cash flow you receive from a Ginnie Mae, brokerage firms make guesses (they say estimates, but I prefer the word guesses) when prepayment will occur. They base these guesses on historical prepayment experience, current and expected future economic environment. One common guess for a 30-year certificate is based on the Public Securities Association (PSA) prepayment benchmark, which assumes prepayment rates will be low for newly originated mortgages and then will speed up as the mortgages become seasoned. Slower or faster prepayment rates are expressed as a percentage of PSA. For example, 150% PSA means one-and-a-half the PSA prepayment rate. The people who make these guesses use a variety of techniques including sophisticated statistical analyses. Sometimes, believe it or not, these guesses are very wrong. My point is that the brokerage firms do not know when prepayment will occur, and neither do you. This is the risk you take.

An Example. Table 1 presents an example of investing in a Ginnie Mae certificate. The analysis assumes that you buy a 30-year (360 months), $100,000 Ginnie Mae certificate with a mortgage rate of 9.5 percent for $105,895. In addition to the PSA assumption, purchase price and terms of the mortgages, the table shows an annual servicing fee of .5%, which goes to the financial institution collecting the monthly mortgage payments.

Table 1
Ginnie Mae Certificate, Projected Cash Flows @ 100% PSA

Face Val	$100,000	PSA	100%	Purchase Price =	($105,895)
Mtg Rate	9.50%	CPR (year)	6% (year)		
Mtg Rate	0.00791667 (month)	Monthly Ser Fee	0.0004167		
Service Fee	0.50% (year)	Term of Loan (months)	360		

Month	Beg. Bal.	CPR	SMM	Mort. Pay.	Sch. Prin.	Interest	Prepay	Ser. Fee	Cash Flow	End. Bal.
1	$100,000.00	0.20%	0.016682%	$840.85	$49.19	$791.67	$16.68	$41.67	$815.87	$99,934.13
2	$99,934.13	0.40%	0.033395%	$840.71	$49.57	$791.15	$33.37	$41.64	$832.45	$99,851.19
3	$99,851.19	0.60%	0.050138%	$840.43	$49.94	$790.49	$50.06	$41.60	$848.89	$99,751.18
4	$99,751.18	0.80%	0.066912%	$840.01	$50.31	$789.70	$66.75	$41.56	$865.19	$99,634.12
5	$99,634.12	1.00%	0.083718%	$839.45	$50.68	$788.77	$83.41	$41.51	$881.35	$99,500.03
6	$99,500.03	1.20%	0.100554%	$838.75	$51.04	$787.71	$100.05	$41.46	$897.34	$99,348.94
7	$99,348.94	1.40%	0.117422%	$837.90	$51.39	$786.51	$116.66	$41.40	$913.16	$99,180.89
8	$99,180.89	1.60%	0.134321%	$836.92	$51.74	$785.18	$133.22	$41.33	$928.81	$98,995.94
18	$96,582.78	3.60%	0.305067%	$819.42	$54.81	$764.61	$294.64	$40.24	$1,073.82	$96,233.33
19	$96,233.33	3.80%	0.322320%	$816.92	$55.08	$761.85	$310.18	$40.10	$1,087.00	$95,868.07
20	$95,868.07	4.00%	0.339605%	$814.29	$55.33	$758.96	$325.57	$39.95	$1,099.92	$95,487.17
21	$95,487.17	4.20%	0.356924%	$811.52	$55.58	$755.94	$340.82	$39.79	$1,112.55	$95,090.77
31	$90,856.12	6.00%	0.514301%	$776.85	$57.57	$719.28	$467.27	$37.86	$1,206.27	$90,331.27
32	$90,331.27	6.00%	0.514301%	$772.85	$57.73	$715.12	$464.57	$37.64	$1,199.79	$89,808.97
33	$89,808.97	6.00%	0.514301%	$768.87	$57.89	$710.99	$461.89	$37.42	$1,193.34	$89,289.19
34	$89,289.19	6.00%	0.514301%	$764.92	$58.04	$706.87	$459.22	$37.20	$1,186.93	$88,771.93
99	$60,333.05	6.00%	0.514301%	$546.93	$69.29	$477.64	$310.29	$25.14	$832.08	$59,953.47
100	$59,953.47	6.00%	0.514301%	$544.11	$69.48	$474.63	$308.34	$24.98	$827.47	$59,575.65
209	$27,330.69	6.00%	0.514301%	$309.81	$93.45	$216.37	$140.56	$11.39	$438.99	$27,096.68
210	$27,096.68	6.00%	0.514301%	$308.21	$93.70	$214.52	$139.36	$11.29	$436.28	$26,863.62
211	$26,863.62	6.00%	0.514301%	$306.62	$93.95	$212.67	$138.16	$11.19	$433.59	$26,631.51
359	$276.69	6.00%	0.514301%	$139.99	$137.80	$2.19	$1.42	$0.12	$141.30	$137.47
360	$137.47	6.00%	0.514301%	$138.56	$137.47	$1.09	$0.71	$0.06	$139.21	($0.71)

Yield (monthly) =	0.68%
Bond-equiv yld (annual) =	8.3%

Beg. Bal. = projected mortgage balance at the beginning of month
SMM = single monthly mortality rate or monthly prepayment rate
Mort. Pay. = projected monthly mortgage payment
Sch. Prin. = projected monthly scheduled principal payment
Interest = projected monthly interest
Prepayment = projected prepayment for the month
Ser. Fee = projected servicing fee for the month
Cash Flow = projected cash flow for month
End. Bal. = projected ending mortgage balance for month

At the beginning of month 1, the certificate has a balance of $100,000. According to the PSA 100% assumption, homeowners will prepay at an annual rate of .2% of the beginning balance in month 1, which converts to a monthly rate of .016682%. Thus, in addition to the monthly mortgage payment of $840.85 (principal of $49.19 and interest of $791.67), you would receive a principal prepayment of $16.68 ($100,000 x .00016682) for a first month total of $857.53. Subtracting out the servicing fee of $41.67 gives you a net cash flow of $815.87 for month 1. At the end of month 1, the mortgage balance of $99,934.13 equals the beginning balance of $100,000 less the sum of the scheduled principal payment of $49.19 (from the amortization schedule) and the principal prepayment of $16.68.

In month 2, the process repeats itself to give you a cash flow of $832.45, and so on for subsequent months. Notice that the constant prepayment rate (CPR) increases from .20% to .40% according to the PSA benchmark assumption before leveling off to 6.00% in month 31. Notice also that your cash flow peaks in month 31 at $1,206.27 and declines to $139.21 in month 360 due to this prepayment assumption. If this investment were a normal mortgage loan without any prepayment of principal, your cash flow would remain constant at $815.87 per month for the entire 360 months.

One final point: given a purchase price of $105,895 (an outflow) and the cash flows you expect to receive over the next 360 months in the next to the last column (inflows), your yield will equal an annual rate of 8.3% (see bond-equivalent yield in the lower right hand corner). The 8.3% anticipated yield assumes two things: (1) a PSA of 100%, and (2) that you can reinvest all your cash flows at 8.3%. Whether you actually receive the 8.3% yield depends on future interest rates in the market that, in turn, impact the reinvestment rate you receive on the cash flows as well as the accuracy of the prepayment assumption. If future interest rates decline and homeowners actually prepay quicker than anticipated, your yield will be less than 8.3% because you will have to reinvest the returned principal at rates below 8.3%. If future interest rates increase and homeowners actually prepay slower than anticipated, your yield will again be less than 8.3% because you will have less cash flow than you assumed you would have to reinvest at the higher market rates.

A Slightly Different Perspective. Instead of determining the yield you expect to receive given the price of the certificate, you can determine the price of the certificate you should pay given your required yield. Looking at it from this perspective, you would be willing to pay $105,985 for the 9.5% coupon certificate in order to receive a yield of 8.3%. Playing some "what if" analysis, Table 2 shows the inverse relationship between the purchase price and required yield for any given PSA prepayment assumption. For example, at a PSA 100% the price of the certificate decreases from $105,985 to $94,521 as the required yield increases from 8.3% to 10.1%. You could also read the table as follows: at a PSA 150% and a required yield of 8.1%, you would be willing to pay $105,985 for the 9.5% coupon certificate. The point is that the price you are willing to pay for a Ginnie Mae certificate depends not only on your required yield, but also on the prepayment assumption.

Table 2 **Relationship Between Purchase Price and Yield**			
Purchase Price = $94,521		**Purchase Price = $105,985**	
PSA	**Yield**	**PSA**	**Yield**
50%	9.6%	50%	8.4%
100%	10.1%	100%	8.3%
150%	10.2%	150%	8.1%
200%	10.4%	200%	8.0%
250%	10.5%	250%	7.9%

Table 2 further shows that prepayments impact investment performance of the pass-through depending on whether you purchase the pass-through at a discount or premium. A discount would exist when current coupon rates, which are driven by current market conditions, exceed the old coupon rate stated on the certificate. Thus, if you purchased the certificate at a discount, you would realize a capital gain at higher actual prepayment rates than anticipated. For example, if you purchased the certificate for $94,521 and the PSA increases from 50% to 250%, your yield increases from 9.6% to 10.5% due to the realized capital gain associated with the quicker prepayment. Think of it this way: purchasing the Ginnie Mae for $94,521, a discount of $5,479, only to have it completely repaid the next day at $100,000 (quite unlikely, I realize, but I say this only to make the point) would give you a capital gain equal to $5,479 on a one-day holding period. On the other hand, if you purchased the certificate at a premium you would realize a capital loss at higher prepayment rates. For example, if you purchased the certificate for $105,985 and the PSA increases from 50% to 250%, your yield declines from 8.4% to 7.9% due to the realized capital loss associated with the quicker prepayment.

III. Collateralized Mortgage Obligations

Overview. As we have seen, the problem with Ginnie Maes is the uncertainty of prepayment risk. Many investors, especially large institutional investors such as pension funds, that need defined cash inflows (income) in order to meet defined cash outflows (pension benefits to retirees) are unwilling to accept this prepayment risk. As a consequence, investment bankers have created CMOs that redirect Ginnie Mae cash flows in order to mitigate the prepayment risk. CMOs, therefore, have more appeal to a broader group of investors than Ginnie Maes. As we shall see, CMOs do not eliminate or even reduce prepayment risk; they only redirect it.

The CMO Structure. A CMO is a class of bonds backed by a pool of Ginnie Mae certificates. In their earliest and simplest form, CMOs were structured with four *sequential-pay* classes or tranches of bonds (A, B, C, and Z) of increasing maturities. The basic idea is rather simple. Investors in the Z tranche or accrual bonds (longest maturities) give up their interest and scheduled principal to investors holding the shortest maturity bonds (tranche A). As long as A bonds are outstanding, investors in B and C bonds receive their stated coupon interest payments, but no principal payments--scheduled or unscheduled. That is, tranche A investors receive all principal payments (scheduled and unscheduled), plus their stated coupon interest plus the Z bond stated coupon interest. As you can see, tranche A investors have more defined cash flows than investors of the other tranches. Thus, tranche A bonds mature the earliest. After retirement of all tranche A bonds, retirement of tranche B bonds occurs the same way followed by retirement of tranche C bonds. After retirement of all the A, B, and C bonds, investors in Z

bonds receive the remaining cash flows until their bonds are retired. Z bonds appeal to investors who wish to mitigate reinvestment risk from coupon and principal payments.

Other CMO Structures. Besides the traditional CMO structure, investment bankers have created several new twists for redirecting Ginnie Mae cash flow. One is called a Planned Amortization Class (PAC) bond. PAC bondholders have priority over all other classes in the CMO issue in receiving principal payments from the Ginnie Mae. In other words, the PAC bondholders have an even greater certainty of receiving cash flow than the traditional A, B and C tranche bondholders. The greater certainty of cash flow for the PAC investors, however, comes at the expense of the non-PAC bondholders who sacrifice their cash flow in order to satisfy the PAC investors. As you can see, CMOs can become rather involved.

Conclusions
Ginnie Mae certificates present you an opportunity for higher yields than what you can achieve with money market instruments, but also higher risks. The uncertainty of homeowners prepaying their mortgages is the main risk. In the final analysis, the yield on a Ginnie Mae certificate depends on future interest rates staying relatively stable. Any movement up or down can exert a detrimental effect on your actual yield depending on whether you bought the certificate at a discount or premium.

CMOs extend the logic of Ginnie Maes by redirecting the pass-through cash flow to other bondholders within the CMO structure. The shortest-maturity bonds (A tranche) are the safest in that the cash flow of these bonds is known with more certainty than the cash flows of other bonds in the structure (B, C and Z tranches). Investors in Z tranche bonds receive their cash flow after the retirement of all the other bonds and, thus, avoid most of the risk associated with reinvestment of coupon interest payments and principal prepayments.

References:
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Business Cycles

Before we leave the equity and real estate notes, I need to say a few words about the business cycle and the movement of interest rates throughout the cycle. Back to equation (1). Dividend growth, which is driven by earnings, is related to the business cycle via the saying "A rising tide lifts all ships." As the economy expands, corporate earnings increase and as the economy contracts, corporate earnings decrease. You would, therefore, think that corporate dividends increase and decrease in a like manner. While this is probably true in terms of a corporation's ability to declare dividends, it is not true with respect to most corporations' actual dividend paying policy. This is because corporate managements tend to be very conservative; that is, they pay stable dividends regardless of earnings. Even at the firm level, actual growth of dividends tends to follow a gradual, step-wise pattern. As a consequence, the business cycle is not much help in evaluating D_1 and g in equation (1).

The business cycle is, however, helpful in evaluating the impact of interest rates (as reflected in k) on the overall stock market. As the economy expands, interest rates tend to increase due to

inflationary pressures. One reason this occurs is because of bottlenecks in the production of goods and services in some industries. For example, an expanding economy may mean the steel industry is operating a full capacity (perhaps 90 percent capacity meaning 90 out of 100 ovens are cooking) even though the automobile industry is operating at 75 percent capacity. In this case, a bottleneck would occur because the steel industry cannot deliver all the steel the auto industry wants. The result is price increases in steel that, in turn, mean higher prices for automobiles. Eventually, consumer spending slows and the economy cools off leading to a decline in the demand for money, a decline in inflationary pressures, and a decline in interest rates. Eventually, the cycle repeats itself.

We know that the prices of stocks respond inversely to increases in interest rates. We also know that the stock market is prospective--it anticipates future events. Thus, higher interest rates associated with an expanding economy leads to a declining stock market even though the economy is still expanding. This is why the stock market is called a leading indicator; it tends to lead both peaks and troughs in the economy by around 9 months, on average. Since the end of World War II, the U. S. economy has experienced an average cycle of around 5 years and the stock market has generally experienced the same 5-year pattern except about 9 months in advance of the economy. The following is a summary of 3 important business cycle and stock market cycle patterns:

1. Stocks tend to lead the business cycle by around 8 months, on average
2. Interest Rates tend to continue rising after the peak of the business cycle
3. The average Business Cycle lasts around 5 years

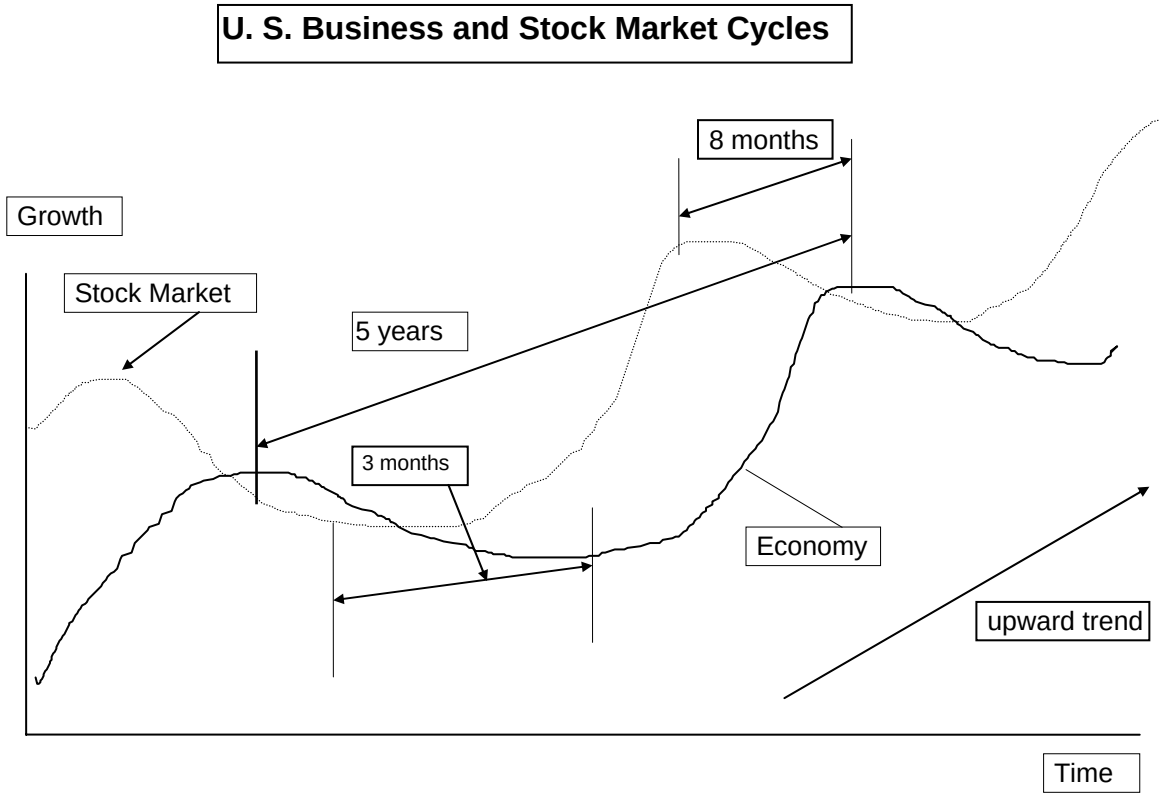
Because the overall market exerts a strong influence on the prices of individual stocks, you need to understand this movement of interest rates throughout the business cycle and the impact it has on the stock market. The graph on the next page is my best attempt to draw these important relationships.

Be sure to recognize that all stocks do not respond to interest rates in the same way. Returns on cyclical stocks (like autos), which are also called value stocks, tend to follow the ups and downs of the business cycle more closely than growth stocks (like high technologies). For definition purposes, growth stocks have dividend growth rates that exceed the average--the average in the U. S. has been around 4% to 5%, historically. Other characteristics of growth stocks include high P/E ratios and high P/Book ratios. Characteristics of value stocks are just the opposite. The table below shows these 3 valuation parameters as of September 12, 1994 relative to their historical averages (source: Barron's).

Parameter	Historical Average	Current Values
P/E Ratio	12-14x	20.5x
P/Book Ratio	2.5-3.0x	3.46x
Dividend Yield	4-4.5%	2.7%

These data suggest that the stock market is near the top of its cycle. Economic data show the economy is operating around 85% capacity with interest rates and inflation increasing. The stock market data combined with the economic data suggest the U. S. is entering the late stage of an

economic expansion and the early stage of a stock market correction (see point A on the graph next page). Of course, this is only one man's opinion and I could be dead wrong.



Introduction to Portfolio Management

Expanding the Efficient Frontier to Multiple Asset Classes. Up to this point, I have focused on stocks in developing the efficient frontier simply as a matter of convenience. In the Capital Asset Pricing Model, however, the market portfolio (M) includes all risky assets, not just stocks. We must, therefore, extend our logic for M to include all other asset classes. The obvious question is, which ones? At this time, you are in a good position to answer this question using the methodology we used before when developing the stock efficient frontier. That is, you would:

1. construct efficient frontiers of all the different asset classes in the world the same way you did for stocks
2. select the one perfectly diversified portfolio off each frontier that lies in the middle of the frontier (the market portfolio for that asset class)
3. check pairwise correlations for all the different asset classes
4. construct the multiple asset efficient frontier by selecting the 2 most uncorrelated asset classes for inclusion on the frontier first, and then proceeding to include the other asset classes in order of decreasing diversification benefits.

This process is graphically presented below. You need to understand this process because we will use the final result, the composition of the market portfolio M, in making the asset allocation decision when constructing the client's optimal portfolio in the next set of study notes. Now this is where things become somewhat fuzzy due to several practical problems. First, how do you define a specific asset class? The answer is that you want to define a particular class to include assets that have similar characteristics (meaning all the assets are highly correlated within the class), but a class that is less than positively correlated with other classes (meaning the class presents positive diversification benefits).

For example, can we define all U. S. domestic stocks as one asset class? From the above definition, the answer to this question is that it depends on the correlations within the U. S. domestic stock market. If we knew that all U. S. stocks moved together, meaning they all act as one security, then we could define this group of stocks as a specific asset class. Likewise, if we knew that all international stocks moved together as a group but moved in a different manner than U. S. stocks (less than positively correlated with U. S. stocks), then we could define international stocks as a separate asset class. In other words, we want to find classes of securities whose returns move together within the group, but differently from other classes of securities. This is a very difficult problem because class boundaries are very vague. For example, is Coca Cola a U. S. stock or an international stock? Coca Cola derives well over 50% of its revenue outside the U. S., but it is domiciled in the U. S. where it pays most of its taxes. Thus, a working definition (but far from perfect) of whether a stock is domestic or international is according to where its home office is located.

Assuming that we can overcome the problem of finding stocks that are highly correlated within a class but relatively uncorrelated across different asset classes, we run into another equally serious problem: that of weighing each asset class within the total market portfolio. In other words, of the total market value of all risky assets in the world, how much does any specific asset class of securities represent relative to the total? The answer to this question is that it depends on measurement of total world wealth. Specifically, we need to add up all the world wealth, divide this wealth into the various asset classes using the correlation analysis we just discussed, and measure the relative weights of each asset class to the total. As you can see, this is an impossible task.

Because of these two huge problems, portfolio managers make educated judgments as to the composition of the market portfolio M. These judgments largely depend on the country in which the client resides. For example, most U. S. clients want U. S. securities to form the base of their portfolio just like most Japanese clients want Japanese securities to form the base of their portfolio. As a consequence, domestic stocks, bonds, and Treasury bills from the country in which the client resides usually comprise the first 3 asset classes for most investors. This is probably not a bad place to begin your asset allocation because these 3 asset classes usually present positive diversification benefits (they are not perfectly positively correlated). To make the discussion manageable, I will take the U. S. investors' perspective from this point forward.

The 4th asset class to include in the typical U. S. portfolio is usually either international equities or real estate. Real estate is probably the better choice since historically it has been less correlated with U. S. stocks than international stocks (see Appendix Table B, Cases in Portfolio Management, AIMR, 1990). International equities would be the 5th asset class and precious metals (usually gold) would be the sixth. I hope you can see the lack of precision in all of this.

Obviously, all U. S. stocks do not move together even though they all may be highly influenced by a common factor, the overall movement of the domestic market. Even this point is debatable in a global capital market where national boundaries are becoming more blurred all the time. Likewise, all international stocks do not move together. My point is that these definitions of different asset classes give us a place to begin the asset allocation process, and I will use these definitions from now on.

Still, we have the second problem of weights. Again, we have a practical solution although it, too, has sever limitations. The solution is to divide the portfolio into a 60/40 split of equities/non-equities where equities include real estate (you could argue that real estate should be in the fixed income class). If you allocate approximately 15% to international equities and 10% to real estate, you would have the remainder of 35% allocated to U. S. domestic equities. Within the non-equity allocation, you could allocate 5% to gold, 5% to cash and the remainder of 30% to U. S. bonds. If you accept these allocations as a working model, the market portfolio (portfolio of risky assets) would look as follows:

Workable Model of the Market Portfolio

Asset Class	**Allocation**
Stocks (U. S.)	35%
Bonds (U. S.)	30%
International Equities	15%
Real Estate (U. S.)	15%
Gold	5%
Total	100%

This model assumes good diversification within each asset class and correlations less than +1 for each possible pairwise combination (i.e., stocks/bonds, stocks/International Equities, bonds/real estate, etc.). It further assumes that these asset classes generate maximum diversification benefits. In other words, no other asset classes, however defined, generate superior diversification benefits. All of these points are highly debatable.